Sequence & Series Review Solutions

1) $u_1 = -7 \quad d = 5 \quad u_{30}?$ $\quad u_n = u_1 + (n-1)d$

$u_{30} = -7 + (30-1)(5) = -7 + (29)(5)$ $\quad \boxed{u_{30} = 138}$

2) $u_1 = 12 \quad d = -4 \quad u_{14}?$

$u_{14} = 12 + (14-1)(-4) = 12 + (13)(-4)$ $\quad \boxed{u_{14} = -40}$

3) $2, 5, 8, 11, \ldots \quad u_1 = 2 \quad d = 3 \quad u_{50}?$

$u_{50} = 2 + (50-1)(3) = 2 + 49(3)$ $\quad \boxed{u_{50} = 149}$

4) $u_6 = 10 \quad u_{10} = 16 \quad u_{25}?$

$u_{10} = u_1 + (10-1)d$

$16 = u_1 + 9d$

$u_6 = u_1 + (6-1)d$

$10 = u_1 + 5d$

$16 = u_1 + 9d$

$-10 = -u_1 - 5d$

$6 = 4d$

$d = 1.5$

$16 = u_1 + 9(1.5)$

$u_1 = 2.5$

Now $u_{25} = u_1 + (25-1)d$

$2.5 + 24(1.5)$ $\quad \boxed{u_{25} = 38.5}$

5) $u_1 = -2 \quad r = 3 \quad u_9?$ $\quad u_n = u_1 \cdot r^{n-1}$

$u_9 = -2(3)^{9-1} = -2(3)^8 =$ $\boxed{-13{,}122 = u_9}$

6) $-3, 1, -\frac{1}{3}, \ldots \quad u_1 = -3 \quad r = -\frac{1}{3} \quad u_7?$

$-3(-1/3)^{7-1} = -3(-1/3)^6 =$ $\boxed{-1/243 = u_7}$

7) $u_1 = 1/4 \quad u_6 = 8 \quad u_{12}?$

$u_6 = u_1 r^{6-1}$

$8 = 1/4\, r^5$

$32 = r^5$ $\quad \sqrt[5]{32} = r$

$r = 2$

$u_{12} = u_1 r^{12-1}$

$u_{12} = (1/4)(2)^{11}$

$\boxed{u_{12} = 512}$

8) $S_{12} \quad u_1 = -6 \quad u_{12} = 10.5 \quad n = 12$

$S_{12} = \frac{n}{2}(u_1 + u_{12}) = \frac{12}{2}(-6 + 10.5) = 6(4.5) = \boxed{27}$

9) $u_1 = 40 \quad d = -3 \quad S_{18} \qquad n/2\,(2u_1 + (n-1)d)$

$18/2\,(2\cdot 40 + (18-1)(-3))$

$9\,(80 + 17(-3)) = 9(80 + -51) = 9(29) = \boxed{261}$

10) $\sum_{i=1}^{25} (2i - 3)$

$u_1 = 2(1) - 3 = -1$

$u_{25} = 2(25) - 3 = 47 \qquad 25/2\,(-1 + 47) = \boxed{575}$

$n = 25$

11) $16 - 8 + 4 - \ldots \quad S_6 \qquad \dfrac{u_1(1 - r^n)}{1 - r} = \dfrac{16(1 - (-1/2)^6)}{1 - (-1/2)}$

$r = -8/16 = -1/2$

$u_1 = 16$

$n = 6$

$= \dfrac{16(.984375)}{1.5} = \boxed{10.5}$

12) $6 + 12 + 24 + \ldots \quad S_7 \qquad \dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{6(2^7 - 1)}{2 - 1} = \dfrac{6(127)}{1} = \boxed{762}$

$r = 12/6 = 2$

$u_1 = 6$

$n = 7$

13) $\sum_{i=1}^{15} 5(-2)^{i-1} \qquad u_1 = 5(-2)^{1-1} = 5(-2)^0 = 5 \longrightarrow r = -2$

$u_2 = 5(-2)^{2-1} = 5(-2)^1 = -10 \qquad n = 15$

$\dfrac{u_1(r^n - 1)}{r - 1} = \dfrac{5((-2)^{15} - 1)}{-2 - 1} = \dfrac{5(-32769)}{-3} = \boxed{54615}$

14) doubles = geometric $\Rightarrow r = 2$

$U_1 = 2000 \qquad U_n = 128{,}000$ (use $U_1 = 2000$ since after $n=1$ it's doubled)

$128{,}000 = 2000\,(2)^{n-1}$ Solve by graphing $\qquad 7 \cdot 30$ min

$64 = (2)^{n-1} \qquad y_1 = 64 \qquad x = 7 \qquad \boxed{\begin{array}{c}210\text{ min}\\ 3.5\text{ hr.}\end{array}}$

$y_2 = 2^{x-1}$

15) $n=1$ = 1 yr of declining $r=.60$ $n=4$

$u_1 = 15{,}000(.60) = \$9000$

$u_4 = u_1(.60)^{n-1} = 9000(.60)^{4-1} = 9000(.6)^3 = \boxed{\$1944}$

16) $\{-20, -9, 2, 13, \ldots 46\}$ How many terms?

What kind of sequence? Adding 11 each time

so... arithmetic $d=11$

$u_n = u_1 + (n-1)d$

$46 = -20 + (n-1)(11)$

$66 = 11n - 11$

$77 = 11n$ $\boxed{n = 7 \text{ terms}}$

17) 14, 16, 18, ...

$n=25$ $d=2$ last row $= u_{25}$

$u_{25} = u_1 + (n-1)(2)$

$u_{25} = 14 + (25-1)(2)$ $u_{25} = \boxed{62 \text{ marchers}}$

$= 14 + 48$

18) $r = 2/3$ $n=1 = 1$ bounce $n=6$

$u_1 = 54(2/3) = 36$ feet

$u_6 = u_1(r)^{n-1} = 36(2/3)^{6-1} = 36(2/3)^5 = \boxed{4.74 \text{ feet}}$

19) $u_1 = 5000$ $d = -250$ $n=15$

Sum! $n/2\,(2u_1 + (n-1)d)$

$15/2(2 \cdot 5000 + (15-1)(-250))$

$7.5(10000 + 14(-250)) = 7.5(6500) = \boxed{\$48{,}750}$

20) $u_1 = 60{,}000$ $r = \uparrow 10\% = 1.10$ Sum $n=10$ n = yrs of sales

$$\frac{60000(1.10^{10}-1)}{1.10-1} = \boxed{\$956{,}245.48}$$

21) $n=20 \Rightarrow 18, 20, 22 \quad r=2$ Sum!

$20/2(2\cdot 18+(20-1)(2))$

$10(36+19(2))$

$\boxed{740 \text{ seats}}$

22) $u_n = 7200$

$u_1 = 5100$ $\quad u_n = u_1 + (n-1)d$

$d = 75$ $\quad 7200 = 5100 + (n-1)(75)$

$2100 = 75n - 75$

$2175 = 75n$

$\boxed{n = 29 \text{ years}}$

23) $u_1 = 3500(.7) = 2450$

$r = .7$ (worth 70% next yr) $\quad u_n = u_1 \cdot r^{n-1}$

$u_n = 1800$ $\quad 1800 = 2450\,(.7)^{n-1}$

$36/49 = .7^{n-1}$

n = # of yrs of declining

Solve by graphing

$y_1 = 36/49$

$y_2 = .7^{x-1}$ $\quad x = 1.86$

$\boxed{2 \text{ yrs}}$

24) $u_1 = 1$

$u_2 = 3$ $\quad d = 2$ $\quad S_n = 1{,}000{,}000$

$u_3 = 5$

$S_n = n/2\,(2u_1 + (n-1)d)$

$1{,}000{,}000 = n/2\,(2\cdot 1 + (n-1)2)$

$2{,}000{,}000 = n(2 + 2n - 2)$

$2{,}000{,}000 = n(2n)$

$2{,}000{,}000 = 2n^2$

$1{,}000{,}000 = n^2$ $\quad n = \boxed{1000 \text{ numbers}}$