IB Math Studies
Topic 1 Day 1: Arithmetic Sequences & Series, Sigma Notation

SEQUENCES & SERIES

- **sequence** a set of quantities arranged in a definite order
- **series** the sum of a sequence
- **arithmetic sequence** a sequence in which each element, after the first, is obtained by adding a fixed number (called the common difference) to the previous element
- **geometric sequence** a sequence in which each element, after the first, is obtained by multiplying the previous element by a fixed number (called the common ratio)

The terms of a sequence are generally labeled $u_1, u_2, u_3, u_4, \ldots\ldots, u_n$.
The n^{th} term of a sequence is labeled u_n.

FORMULA to find the n^{th} term

The nth term of an arithmetic sequence: $u_n = u_1 + (n-1)d$ *IB FORMULA SHEET*

$u_n = n^{th}$ term u_1 = first term n = # of terms d = common difference

ex. (a.) Is the sequence 9, 12, 15, 18, ... an arithmetic sequence?
(b.) Find the 8^{th} term for the sequence described in part (a.).

yes $d = 12-9 = 3$

$u_1 = 9$ $d = 3$ $u_n = u_1 + (n-1)d$ $u_8 = 9 + (8-1)3$ $u_8 = 9+7(3)$ $u_8 = 30$

ex. The 15^{th} term of an arithmetic sequence is 45. If the first term is –11, find the common difference.

$u_1 = -11$ $u_{15} = 45$ $u_n = u_1 + (n-1)d$
$45 = -11 + (15-1)d$
$45 = -11 + 14d$
$56 = 14d$
$d = 3$

ex. For the sequence 15, 11, 7, ... find the 20^{th} term.

$u_1 = 15$ $d = -4$ $u_n = u_1 + (n-1)d$
$u_{20} = 15 + (20-1)(-4)$
$u_{20} = 15 + 9(-4)$
$u_{20} = -61$

ex. An arithmetic sequence has a 7^{th} term of 16.5 and a 12^{th} term of 24. Find the 24^{th} term.

*this is different from how the book does it ... but I think it's better.

$u_7 = 16.5 \Rightarrow u_7 = u_1 + (7-1)d \Rightarrow 16.5 = u_1 + 6d$
$u_{12} = 24 \Rightarrow u_{12} = u_1 + (12-1)(d) \Rightarrow 24 = u_1 + 11d$

Solve system!
$24 = u_1 + 11d$
$-16.5 = -u_1 - 6d$
$7.5 = 5d$
$d = 1.5$

now find u_1
$16.5 = u_1 + 6(1.5)$
$u_1 = 7.5$

now find u_{24}
$u_{24} = 7.5 + (24-1)(1.5)$
$u_{24} = 42$

ex. A car whose original value was \$25600 decreases in value by \$90 per month. How long will it take before the car's value falls below \$15000?

In this case the price of the car decreases by a constant \$90 a month.
So this is arithmetic with $d=-90, u_1=25600, u_n=15000$

$15000 = 25600 + (n-1)(-90)$
$-10,600 = -90n + 90$ → $-10690 = -90n$
$n = 118.8$

$n = 119$ months

GRAPHICS CALCULATOR & SEQUENCES
Most graphics calculators have an automatic memory facility (call **Ans**) that stores the result of the last calculation as well as an ability to remember the actual calculation. This can be very useful in listing a sequence.

For example, to list the sequence 5, 12, 19, 26, ... using your calculator:
STEP 1: Type the first term into the calculator, 5, and hit ENTER.
STEP 2: Type +7, which means "add 7 to the previous answer" and hit ENTER
STEP 3: Continue to hit ENTER to obtain each successive term in the sequence

SEQUENCE MODE on the TI CALCULATOR
The TI calculator has a sequence mode that changes the Y= screen to a sequence version. To do this go to MODE and switch to Seq mode.

There are three sequence forms: u(n), v(n), and w(n) which can be accessed on the Y= screen. Once these equations are defined, it is possible to find the nth terms using the calculator.

ex. Let $u_n = 127 - 7n$. Find the 15^{th} term.

To use sequence mode, go to MODE and switch to Seq. Then go to y= and for your *n*Min put 1.
Then for u_n put $127 - 7n$. Note, the button X, T, , n will now give an "n" when you hit it.

Method 1: USING THE u(n) command to find the nth term:
In the Y= screen, let u(n) = 127 – 7n
From the home screen use the 2^{nd} function of key "7" to call up u.
Type then type (15) using parentheses, to find the 15^{th} term of the sequence. So the screen will say u(15) = 22

Method 2: USING A TABLE to find nth terms:
In the Y= screen, let u(n) = 127 – 7n
Go to TBL SET to set-up your n to start at 1 and go by 1's.
Then go to the table to see the nth term and its corresponding value. u(15) = 22

Method 3: USING seq(
NOTE: You do NOT have to be in sequence mode to use this
Go to 2^{nd} STAT, choose OPS, pick 5: seq(
Use the following format at the home screen seq(127 – 7X, X, 1, 15)
OR Expr: 127 – 7X, Var: X, start: 1, end: 15, step: 1 and then go to Paste
This will list all of the elements in the sequence from the1st to 15^{th} term
(** NOTE, if you only want the 15^{th} term then type in 15, 15 or enter 15 as your start and end)

To use your calc to find a sum...

Go to 2^{nd} STAT, choose MATH then pick 5: sum(
Then follow Method 3 above. It will add up all 15 terms.
The answer should be 1065.

So your calc will end up saying sum(seq(

Arithmetic Sequences & Series, Sigma Notation

ARITHMETIC SERIES: If the terms of a sequence are added together, the result is known as series.

The sequence: 1, 2, 3, 4, 5, 6, ...
gives the series: 1 + 2 + 3 + 4 + 5 + 6 + ...

The sum of the terms of a sequence is referred to as S_n, the sum of n terms of a series.
$S_n = u_1 + u_2 + u_3 + u_4 + ...$

FORMULA to find the Sum

I know the last term

The sum of the n terms of an Arithmetic sequence: $S_n = \frac{n}{2}[2u_1 + (n-1)d] = \frac{n}{2}(u_1 + u_n)$

I don't know the last term.

IB FORMULA SHEET

$u_n = n^{th}$ term u_1 = first term n = # of terms d = common difference

ex. Find the sum of 20 terms of the series –2 + 1 + 4 + 7 + 10 + ...

I don't know u_{20} so use the 1st formula $n=20, u_1=-2, d=3$

$\frac{20}{2}[2(-2)+(20-1)(3)] = 10[-4+19(3)] = 10[53] = \boxed{530}$

ex. Find the sum of $10+8+6+\ldots-16$.

I don't know how many terms there are.
I do know $u_1 = 10$ $d = -2$ $u_n = -16$

$u_n = u_1 + (n-1)d$
$-16 = 10 + (n-1)(-2)$
$-26 = -2n + 2$
$n = 14$

2nd formula
now $n=14$ $u_1=10$ $u_{14}=-16$

$\frac{14}{2}(10 + -16) = 7(-6) = \boxed{-42}$

ex. A new business is selling home computers. They predict that they will sell 20 computers in their first month, 23 in the second month, 26 in the third month and so on, in arithmetic sequence. How many months will pass before the company expects to sell their thousandth computer?

This is a sum question. Why? 1st month = 20 computers sold
2nd month = 23
43

So ... $u_1 = 20$ $d = 3$ $S_n = 1000$ $n = ?$

$1000 = \frac{n}{2}[2(20) + (n-1)(3)]$
$2000 = n[40 + 3n - 3]$
$2000 = n[37 + 3n]$
$2000 = 37n + 3n^2$
$0 = 3n^2 + 37n - 2000$

use calc. to solve
$n = 20.4$ or -32.7
21 months

Arithmetic Sequences & Series, Sigma Notation

SIGMA NOTATION

$\sum$ stands for "the sum of...."

This means that the expression $\sum_{i=1}^{n} u_i = u_1 + u_2 + u_3 + \ldots\ldots + u_{n-1} + u_n$

For example: $\sum_{i=1}^{5} 2+i = (2+1) + (2+2) + (2+3) + (2+4) + (2+5) = 3 + 4 + 5 + 6 + 7 = 25$

ex. Find $\sum_{i=1}^{5} 5+2i$

method 1: Find all 5 terms and add

$i=1 \quad 5+2(1)=7$

$i=2 \quad 5+2(2)=9$

$i=3 \quad 5+2(3)=11$

$i=4 \quad 5+2(4)=13$

$i=5 \quad 5+2(5)=15$

Sum: $7+9+11+13+15$

$\boxed{55}$

method 2: use $S_n = \frac{n}{2}(u_1+u_n)$

$n=5$ ($\sum_{i=1}^{5}$ means $u_1 \to u_5$ which is 5 terms)

$u_1 = 7$ (see above)

$u_n = u_5 = 15$ (see above)

$S_n = \frac{5}{2}(7+15) = 2.5(22) = \boxed{55}$

This method is good when n is large!