

# IB Math Standard Level Summer Assignment 2017

## QUADRATICS

Section A: Factor each quadratic. If the quadratic cannot be factored, write "prime."

1.  $x^2 - x - 2$

$$(x-2)(x+1)$$

2.  $x^2 + 3x - 4$

$$(x+4)(x-1)$$

3.  $8x^2 - 50y^2$

$$2(4x^2 - 25y^2)$$

$$2(2x+5y)(2x-5y)$$

4.  $3x^2 - 5x + 2$      2.3=6

6	-5
-6, 1	
-3, -2	

$$3x^2 - 3x - 2x + 2$$

$$3x(x-1) - 2(x-1)$$

$$(3x-2)(x-1)$$

5.  $2x^2 - x - 6$      -12 | -1

-12	-1
-4, 3	

$$2x^2 - 4x + 3x - 6$$

$$2x(x-2) + 3(x-2)$$

$$(2x+3)(x-2)$$

6.  $x^3 - 3x^2 - 18x$

$$x(x^2 - 3x - 18)$$

$$x(x-6)(x+3)$$

Section B: Solve each equation using any method except graphing or guess and check.

1.  $x^2 + 25 = 10x$

$$x^2 - 10x + 25 = 0$$

$$(x-5)(x-5) = 0$$

$$x-5 = 0 \quad x-5 = 0$$

$$x = 5$$

2.  $x^2 + 3x - 1 = 0$

$$\frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$$

won't factor!  
a=1 b=3 c=-1

$$\frac{-3 \pm \sqrt{9+4}}{2} = \frac{-3 \pm \sqrt{13}}{2} \approx 0.303, -3.30$$

3.  $x + \frac{12}{x} = 7$

$$x^2 + 12 = 7x$$

$$x^2 - 7x + 12 = 0$$

$$(x-4)(x-3) = 0$$

$$x = 4 \quad x = 3$$

4.  $x^2 + 2 = 9$

$$x^2 = 7$$

$$x = \pm \sqrt{7}$$

$$x \approx \pm 2.65$$

5.  $x^2 - 5x = 0$

$$x(x-5) = 0$$

$$x = 0 \quad x - 5 = 0$$

$$x = 5$$

6.  $36x^2 - 25 = 0$

$$(6x+5)(6x-5) = 0$$

$$6x+5 = 0 \quad 6x-5 = 0$$

$$x = -5/6 \quad x = 5/6$$

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Section C: State the following for each of the given equations: axis of symmetry, vertex, direction of opening, x-intercepts, and y-intercepts. Then sketch the graph using that information.

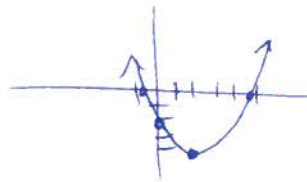
1.  $y = -2(x + 2)(x - 1)$

Opens down ( $a = -2$ )  
 x-int:  $x + 2 = 0$   $x - 1 = 0$   
 $x = -2, 1$   
 y-int:  $-2(0 + 2)(0 - 1) = y$   
 $y = 4$   
 Axis/vertex:  
 $y = -2(x^2 + x - 2)$   
 $y = -2x^2 - 2x + 4$   
 axis  $x = \frac{-(-2)}{2(-2)} = 2/-4$   
 $x = -1/2$   
 vertex:  $y = -2(-1/2)^2 - 2(-1/2) + 4$   
 $(-1/2, 4.5)$



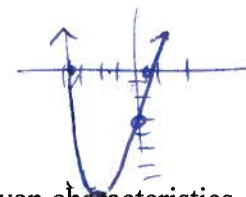
2.  $y = 0.5(x - 2)^2 - 4$

Opens up ( $a = 0.5$ )  
 vertex:  $(2, -4)$   
 axis:  $x = 2$   
 y-int:  $0.5(0 - 2)^2 - 4$   
 $y = -2$   
 x-int:  $0 = 0.5(x - 2)^2 - 4$   
 $4 = 0.5(x - 2)^2$   
 $8 = (x - 2)^2$   
 $x - 2 = \sqrt{8}$   $x - 2 = -\sqrt{8}$   
 $x = 4.83, -0.828$



3.  $y = 2x^2 + 6x - 3$

Opens up ( $a = 2$ )  
 axis:  $x = \frac{-6}{2(2)} = -6/4$   
 $x = -3/2$   
 vertex:  $2(-3/2)^2 + 6(-3/2) - 3$   
 $(-3/2, -7.5)$   
 y-int  $y = 2(0)^2 + 6(0) - 3$   
 $y = -3$   
 x-int:  $0 = 2x^2 + 6x - 3$   
 QF  $x \approx .436$   
 $x \approx -3.44$



Section D: Find the values of  $p$  such that the equations below have the given characteristics.

Hint: Use the discriminant.

$b^2 - 4ac$

1. Two different real roots

$px^2 + 5x + 2 = 0$   
 $\rightarrow D > 0$   $a = p$   
 $b = 5$   
 $c = 2$   
 $5^2 - 4(p)(2) > 0$   
 $25 - 8p > 0$   
 $-8p > -25$   
 $p < 25/8$   
 or  $p < 3.125$

2. Two equal real roots

$2x^2 - 3x + p = 0$   
 $D = 0$   $a = 2$   
 $b = -3$   
 $c = p$   
 $(-3)^2 - 4(2)(p) = 0$   
 $9 - 8p = 0$   
 $-8p = -9$   
 $p = 9/8$   
 or  $p = 1.125$

3. No real roots

$px^2 - 4px + 5 - p = 0$   
 $D < 0$   $a = p$   
 $b = -4p$   
 $c = 5 - p$   
 $(-4p)^2 - 4(p)(5 - p) < 0$   
 $16p^2 - 20p + 4p^2 < 0$   
 $20p^2 - 20p < 0$   
 $20p(p - 1) < 0$   
 $20p < 0$   $p - 1 > 0$  or  $20p > 0$   $p - 1 < 0$   
 $p < 0$  and  $p > 1$  no solution  $p > 0$  and  $p < 1$

Section E: Use your graphing calculator to find the following

1. Solve  $3x^2 - x - 5 = 0$

$x = -1.14, x = 1.47$

2. Intersection points of  $y = -x^2 - 5x + 3$   
 and  $y = x^2 + 3x + 11$

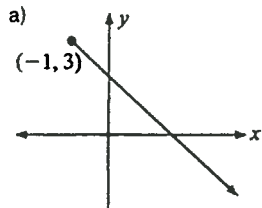
$(-2, 9)$

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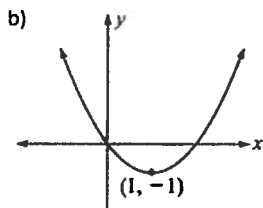
## FUNCTIONS

Section A: For each of the following find the domain and range without using a calculator.

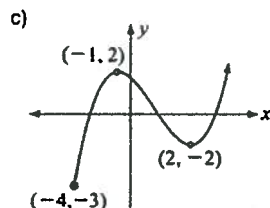
1.



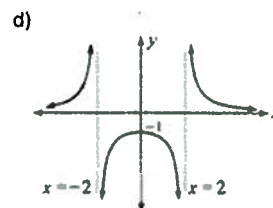
D:  $[-1, \infty)$   
R:  $(-\infty, 3]$



D:  $\mathbb{R}$  or  $(-\infty, \infty)$   
R:  $y \geq -1$  or  $[-1, \infty)$



D:  $[-4, \infty)$   
R:  $[-3, \infty)$



D:  $\mathbb{R} \setminus \{-2, 2\}$   
R:  $(0, \infty) \cup (-\infty, -1]$

2.

a)  $f(x) = \sqrt{x}$

D:  $[0, \infty)$  or  $x \geq 0$   
R:  $[0, \infty)$  or  $y \geq 0$

b)  $f(x) = \sqrt{4-x}$

$4-x \geq 0$   
 $-x \geq -4$   
 $x \leq 4$   
D:  $(-\infty, 4]$   
R:  $[0, \infty)$

c)  $y = 5x - 3x^2$

D:  $\mathbb{R}$   
Vertex:  $x = \frac{-5}{2(-3)}$   
 $y = 5(5/6) - 3(5/6)^2$   
 $y = 2\frac{1}{2}$   
R:  $(-\infty, 2\frac{1}{2}]$

d)  $y = \frac{x+4}{x-2}$

D:  $\mathbb{R} \setminus \{2\}$   
R:  $\mathbb{R} \setminus \{1\}$

Section B: Find the inverse of each function.

1.  $f(x) = 2x + 1$

$x = 2y + 1$

$x - 1 = 2y$

$\frac{x-1}{2} = y$  or  $\frac{x-1}{2} = f^{-1}(x)$

2.  $f(x) = \frac{x^2}{3}$

$x = \frac{y^2}{3}$

$3x = y^2$

$y = \pm \sqrt{3x}$

3.  $g(x) = \frac{5}{x-2}$

$x = \frac{5}{y-2}$

$x(y-2) = 5$

$y-2 = 5/x$

$y = \frac{5}{x} + 2$

4.  $g(x) = \sqrt{4-x} + 1$

$x = \sqrt{4-y} + 1$

$x-1 = \sqrt{4-y}$

$(x-1)^2 = 4-y$

$(x-1)^2 - 4 = -y$

$-(x-1)^2 + 4 = y$

5. If the point (2, 7) is on the graph of  $f(x)$ , what point must be on the graph of  $f^{-1}(x)$ ?

(7, 2)

\* all x's + y's are switched on inverses!

6. Explain, in complete sentences, the relationship between a function and its inverse.

reflection over the line  $y=x$

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Section C: Let  $f(x) = 2x^2 - 1$ ;  $g(x) = 3x$  and  $h(x) = 5 - x$ . Find the following.

1.  $f(-3)$

$$2(-3)^2 - 1$$

$$2(9) - 1 = \boxed{17}$$

2.  $(f \circ g)(x)$

$$2(3x)^2 - 1$$

$$2 \cdot 9x^2 - 1$$

$$\boxed{18x^2 - 1}$$

3.  $(h \circ f)(x)$

$$5 - (2x^2 - 1)$$

$$5 - 2x^2 + 1$$

$$\boxed{-2x^2 + 6}$$

4.  $(f \circ h)(x + 1)$

$$h(x+1) = 5 - (x+1) = 5 - x - 1$$

$$4 - x$$

$$f(4-x) = 2(4-x)^2 - 1$$

$$2(16 - 8x + x^2) - 1$$

$$32 - 16x + 2x^2 - 1 = \boxed{2x^2 - 16x + 31}$$

5.  $(g \circ h)(4)$

$$h(4) = 5 - 4 = 1$$

$$g(1) = 3(1)$$

$$\boxed{3}$$

6.  $(f \circ f)(-1)$

$$f(-1) = 2(-1)^2 - 1 = 1$$

$$f(1) = 2(1)^2 - 1$$

$$\boxed{1}$$

Section D: Answer the following questions concerning equations of lines

1. What is the slope, x-intercept, and y-intercept of the equation  $5x - 4y = 8$ ?

x-int:  $5x - 4(0) = 8$

$$5x = 8$$

$$\boxed{x = 8/5}$$

y-int:  $5(0) - 4y = 8$

$$-4y = 8$$

$$\boxed{y = -2}$$

Slope:  $5x - 8 = 4y$

$$5/4x - 2 = y$$

$$\boxed{m = 5/4}$$

2. What is the slope-intercept form of the equation of the line between the points (4, 3) and (7, -2)?

$$m = \frac{3 - (-2)}{4 - 7} = \frac{5}{-3}$$

$$m = -5/3$$

$$y - 3 = -5/3(x - 4)$$

$$y - 3 = -5/3x + 20/3$$

$$y = -5/3x + 20/3 + 9/3$$

$$\boxed{y = -5/3x + 29/3}$$

3. What is the slope-intercept form of a line perpendicular to  $y = -2x + 9$  passing through the (4, 7)?

$$y = -2x + 9$$

$$m = -2$$

$$\perp m = 1/2$$

$$y - 7 = 1/2(x - 4)$$

$$y - 7 = 1/2x - 2$$

$$\boxed{y = 1/2x + 5}$$

Section E: Find the horizontal & vertical asymptotes and holes (if applicable) of the following.

1.  $y = \frac{1}{2x-5}$

VA:  $2x - 5 = 0$

$$x = 5/2$$

HA:  $y = 0$

2.  $y = \frac{x^2-5}{2x^2-12}$

VA:  $2x^2 - 12 = 0$

$$2x^2 = 12$$

$$x^2 = 6$$

$$x = \pm\sqrt{6}$$

HA:  $y = 1/2$

3.  $y = \frac{x^2+2x-3}{x^3+6x^2-7x} = \frac{(x+3)(x-1)}{x(x+7)(x-1)}$

Hole:  $x - 1 = 0$

$$x = 1$$

VA:  $x = 0$   $x + 7 = 0$

$$x = -7$$

HA:  $y = 0$

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Section F: For each pair of functions  $f(x)$  and  $g(x)$ , describe the transformations that would transform  $f(x)$  into  $g(x)$ .

1.  $f(x) = x^2$ ;  
 $g(x) = (x - 5)^2 + 2$

right 5  
 up 2

2.  $f(x) = \sqrt{x}$ ;  
 $g(x) = \sqrt{3x} - 10$

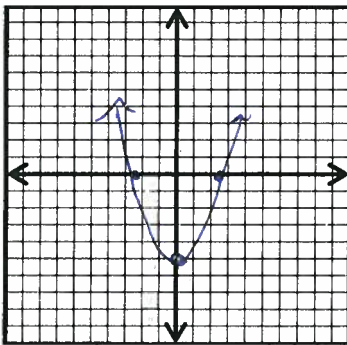
Hor. compress  $\frac{1}{3}$   
 Down 10

3.  $f(x) = e^x$ ;  
 $g(x) = -5(e)^{x-1}$

Right 1  
 Flip vertically over x-axis  
 Ver. stretch of 5

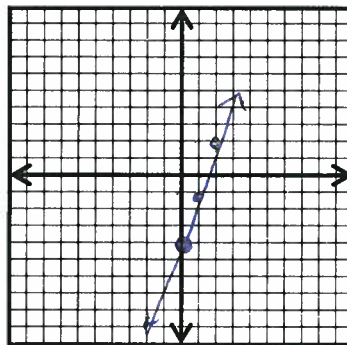
Section G: Graph each function, clearly showing its key features (maxima, minima, and intercepts). Identify its domain and range. (Remember: No calculator!)

1.  $f(x) = x^2 - 5$



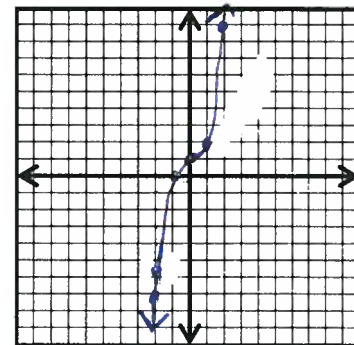
D:  $\mathbb{R}$   
 R:  $[-5, \infty)$

2.  $f(x) = 3x - 4$



D:  $\mathbb{R}$   
 R:  $\mathbb{R}$

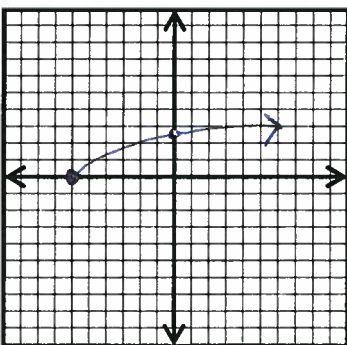
3.  $f(x) = x^3 + 1$



D:  $\mathbb{R}$   
 R:  $\mathbb{R}$

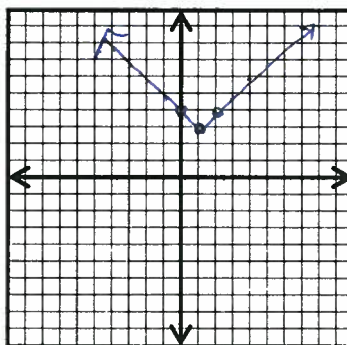
x	y
-2	-9
-1	0
0	1
1	2
2	9

4.  $f(x) = \sqrt{x+6}$



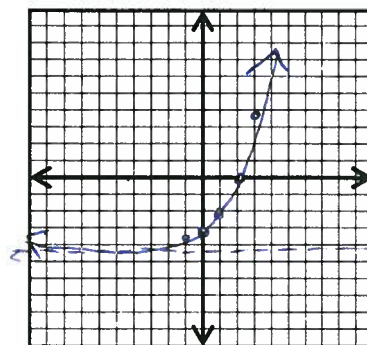
D:  $[-6, \infty)$   
 R:  $[0, \infty)$

5.  $f(x) = |x - 1| + 3$



D:  $\mathbb{R}$   
 R:  $[3, \infty)$

6.  $f(x) = 2^x - 4$



D:  $\mathbb{R}$   
 R:  $(-4, \infty)$

x	y
-1	-3.5
0	-3
1	-2
2	0
3	4



# IB Math Standard Level Summer Assignment 2017

## ALGEBRA

Section A: Simplify the following without a calculator.

<p>1. <math>(2x^5)^{-3}</math>  <math>\frac{1}{(2x^5)^3} = \frac{1}{8x^{15}}</math></p>	<p>2. <math>8^{2/3}</math>  <math>(\sqrt[3]{8})^2 = 2^2 = 4</math></p>	<p>3. <math>81^{-3/4}</math>  <math>\frac{1}{(\sqrt[4]{81})^3} = \frac{1}{3^3} = \frac{1}{27}</math></p>	<p>4. <math>\sqrt[3]{16x^3} = \sqrt[3]{8 \cdot 2x^3}</math>  <math>2x\sqrt[3]{2}</math></p>
<p>5. <math>\sqrt{10x^2} \cdot \sqrt{70x^6}</math>  <math>\sqrt{700x^8}</math>  <math>\sqrt{100 \cdot 7x^8} = 10x^4\sqrt{7}</math></p>	<p>6. <math>\frac{\sqrt{72x^4}}{\sqrt{3x}} = \sqrt{24x^3}</math>  <math>\sqrt{4 \cdot 6 \cdot x^2 x}</math>  <math>2x\sqrt{6x}</math></p>	<p>7. <math>\frac{5}{7-\sqrt{5}} \cdot \frac{7+\sqrt{5}}{7+\sqrt{5}}</math>  <math>\frac{35+5\sqrt{5}}{49+7\sqrt{5}-7\sqrt{5}-5} = \frac{35+5\sqrt{5}}{44}</math></p>	<p>8. <math>\sqrt{5} - 5\sqrt{125} - 7\sqrt{180}</math>  <math>\sqrt{5} - 5\sqrt{25 \cdot 5} - 7\sqrt{36 \cdot 5}</math>  <math>\sqrt{5} - 5 \cdot 5\sqrt{5} - 7 \cdot 6\sqrt{5}</math>  <math>1\sqrt{5} - 25\sqrt{5} - 42\sqrt{5}</math>  <math>-66\sqrt{5}</math></p>

Section B: Solve using algebra.

<p>1. <math>3x + 7y = 36</math>  <math>x = 5y - 10</math>  <math>3(5y-10) + 7y = 36</math>  <math>15y - 30 + 7y = 36</math>  <math>22y = 66</math>  <math>y = 3</math>  <math>x = 5(3) - 10</math>  <math>x = 5</math></p>	<p>2. <math>6x + 10y = 32</math>  <math>4x - 2y = 4</math>  <math>6x + 10y = 32</math>  <math>20x - 10y = 20</math>  <math>\frac{26x = 52}{x = 2}</math>  <math>6(2) + 10y = 32</math>  <math>12 + 10y = 32</math>  <math>10y = 20</math>  <math>y = 2</math></p>
<p>3. <math>x = y^2</math>  <math>x - y = 6</math>  <math>y^2 - y = 6</math>  <math>y^2 - y - 6 = 0</math>  <math>(y-3)(y+2) = 0</math>  <math>y = 3, y = -2</math>  <math>x = (3)^2 = 9</math>  <math>x = (-2)^2 = 4</math>  <math>(9, 3)</math>  <math>(4, -2)</math></p>	<p>4. <math>x^2 + y^2 = 25</math>  <math>y = x^2 - 13</math>  <math>y + 13 = x^2</math>  <math>y^2 + y - 12 = 0</math>  <math>(y+4)(y-3) = 0</math>  <math>y = -4</math>  <math>y = 3</math>  <math>x^2 + (-4)^2 = 25</math>  <math>x^2 + 16 = 25</math>  <math>x^2 = 9</math>  <math>x = \pm 3</math>  <math>(\pm 3, -4)</math>  <math>x^2 + (3)^2 = 25</math>  <math>x^2 + 9 = 25</math>  <math>x^2 = 16</math>  <math>x = \pm 4</math>  <math>(\pm 4, 3)</math></p>

Section C: Solve for x. Eliminate any extraneous solutions, if any.

<p>1. <math>\sqrt{37-3x} = x-3</math>  <math>37-3x = x^2-6x+9</math>  <math>0 = x^2-3x-28</math>  <math>(x-7)(x+4) = 0</math>  <math>x = 7, x = -4</math>  <math>x = 7</math></p>	<p>2. <math>-3(2x+1)^3 = -192</math>  <math>(2x+1)^3 = 64</math>  <math>2x+1 = 4</math>  <math>2x = 3</math>  <math>x = 3/2</math></p>	<p>3. <math>\frac{x}{3} - \frac{5}{2} = \frac{-3}{x} \Rightarrow \frac{2x}{6} - \frac{15}{6} = \frac{-3}{x}</math>  <math>\frac{2x-15}{6} = \frac{-3}{x}</math>  <math>2x^2 - 15x = -18</math>  <math>2x^2 - 15x + 18 = 0</math>  <math>(2x-3)(x-6) = 0</math>  <math>x = 3/2, x = 6</math></p>
<p>4. <math>\frac{4x-1}{x+1} = \frac{x-1}{1}</math>  <math>4x-1 = x^2-1</math>  <math>0 = x^2-4x</math>  <math>0 = x(x-4)</math>  <math>x = 0, x = 4</math></p>	<p>5. <math>2 3x-1  + 5 = -2x+8</math>  <math>2 3x-1  = -2x+3</math>  <math> 3x-1  = -x+3/2</math>  <math>3x-1 = -x+3/2</math>  <math>4x = 5/2</math>  <math>x = 5/8</math></p>	<p>6. <math>5(x-3) \leq 8(x+5)</math>  <math>5x-15 \leq 8x+40</math>  <math>-3x \leq 55</math>  <math>x \geq -55/3</math></p>

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## SEQUENCES AND SERIES

Section A: Answer the following questions concerning arithmetic sequences and series.

1. Consider the sequence 87, 83, 79, 75...  $u_1 = 87$   $d = -4$

a) What is the formula for the general term  $u_n$ ?

$$u_n = 87 + (n-1)(-4)$$

or

$$u_n = 87 - 4n + 4$$

$$u_n = -4n + 91$$

b) What is the 40<sup>th</sup> term?

$$u_n = -4(40) + 91$$

$$u_n = -69$$

c) Is -143 a member?

$$-143 = -4n + 91$$

$$-234 = -4n$$

$$n = 58.5$$

**no!**

d) What is the sum of the first 22 terms?

$$S_{22} = \frac{n}{2}(2u_1 + (n-1)d)$$

$$= \frac{22}{2}(2(87) + (22-1)(-4))$$

$$S_{22} = 990$$

2. A sequence is defined by  $u_n = 3n - 2$

a) What is  $u_1$  and  $d$ ?

$$u_1 = 3(1) - 2$$

$$u_1 = 1$$

$$u_2 = 3(2) - 2$$

$$u_2 = 4$$

$$d = 3$$

b) What is the 57<sup>th</sup> term?

~~$$u_n = 1 + (n-1)(3)$$~~
~~$$u_n = 1 + 3n - 3$$~~
~~$$u_n = 3n - 2$$~~

$$u_{57} = 3(57) - 2$$

$$u_{57} = 169$$

c) What is the first term to exceed 450?

$$450 = 3n - 2$$

$$452 = 3n$$

$$n = 150.\bar{6}$$

#

$$u_{151}$$

d) What is the sum of the first 57 terms?

$$S_{57} = \frac{n}{2}(u_1 + u_{57})$$

$$= \frac{57}{2}(1 + 169)$$

$$28.5(170)$$

$$S_{57} = 4845$$

3) Find the general term  $u_n$  for an arithmetic sequence given that  $u_7 = 41$  and  $u_{13} = 77$ .

$$(7, 41) \quad (13, 77)$$

$$d = m = \frac{41 - 77}{7 - 13}$$

$$= \frac{-36}{-6}$$

$$d = 6$$

$$u_n = u_1 + (n-1)d$$

$$u_7 = u_1 + (7-1)d$$

$$41 = u_1 + (7-1)(6)$$

$$u_1 = 5$$

$$u_n = 5 + (n-1)(6)$$

$$5 + 6n - 6$$

$$u_n = 6n - 1$$

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Section B: Answer the following questions concerning arithmetic sequences and series.

1. Consider the sequence 12, -6, 3, -1.5, ...  $u_1 = 12$   $r = -1/2$

- a) What is the formula for the general term  $u_n$ ?    b) What is the 13<sup>th</sup> term?    c) What is the sum of the first 10 terms?    d) What is the infinite sum of the sequence?

$$u_n = 12 \cdot (-1/2)^{n-1}$$

$$u_{13} = 12 \cdot (-1/2)^{13-1}$$

$$S_n = \frac{12((1/2)^{10} - 1)}{(-1/2 - 1)}$$

Since  $|r| < 1$   
This is defined

$$u_{13} = \frac{3}{1024}$$

$$\frac{1023}{128}$$

$$S_\infty = \frac{12}{(1 - (-1/2))}$$

$$\boxed{8}$$

2. Find the general term  $u_n$  for an arithmetic sequence given that  $u_4 = 24$  and  $u_7 = 192$ .

$$\begin{array}{cccc} & \cdot r & \cdot r & \cdot r \\ 24 & \xrightarrow{\quad} & \xrightarrow{\quad} & \xrightarrow{\quad} 192 \\ u_4 & & u_5 & u_6 & u_7 \end{array}$$

$$u_4 = u_1 \cdot r^{4-1}$$

$$24 = u_1 \cdot (2)^3$$

$$u_1 = 3$$

$$u_n = 3 \cdot (2)^{n-1}$$

$$24 \cdot r^3 = 192$$

$$r^3 = 8 \quad r = 2$$

3. In 1998 there were 3000 koalas on Koala Island. Since then, the population of koalas on the island has increased by 5% each year. How many koalas were on the island in 2001? In what year will the population first exceed 5000?

$$u_1 = 3000$$

$$r = 1.05 \text{ (}\uparrow 105\% \text{)}$$

$$n = 4$$

$$u_4 = 3000(1.05)^{4-1}$$

$$u_4 = 3472.9$$

$$\boxed{3472 \text{ Koalas}}$$

$$5000 = 3000(1.05)^{n-1}$$

$$5/3 = 1.05^{n-1}$$

$$\log(5/3) = (n-1) \log 1.05$$

$$\log(5/3) \div \log(1.05) = n-1$$

$$n = 11.5 \sim \text{during yr 11}$$

$$\boxed{2008}$$

Section C: Find the following sums written in Sigma Notation.

1.  $\sum_{r=1}^4 (3r - 5)$

$$u_1 = 3(1) - 5 = -2$$

$$u_2 = 3(2) - 5 = 1$$

$$u_3 = 3(3) - 5 = 4$$

$$u_4 = 3(4) - 5 = 7$$

$$\boxed{\text{Sum} = 10}$$

2.  $\sum_{i=1}^{15} 50(0.8)^{i-1}$

$$u_1 = 50(0.8)^0 = 50$$

$$u_2 = 50(0.8)^1 = 40$$

$$u_3 = 50(0.8)^2 = 32$$

Geometric Seq.

$$u_1 = 50 \quad n = 15 \quad r = 0.8$$

$$\frac{50((0.8)^{15} - 1)}{(0.8 - 1)}$$

$$\approx 241.2$$



# IB Math Standard Level Summer Assignment 2017

## EXPONENTIAL AND LOGARITHMIC EQUATIONS

Section A: Find the following without using a calculator.

1.

a) $\log_4 64$	b) $\log_2 1/4$	c) $\log_8 1$	d) $\log_9 3$	e) $\log_m m^6$	f) $\ln(e^{2x})$
$4^x = 64$	$2^x = 1/4$	$8^x = 1$	$9^x = 3$	$m^x = m^6$	$e^? = e^{2x}$
<span style="border: 1px solid black; padding: 2px;">3</span>	<span style="border: 1px solid black; padding: 2px;">-2</span>	<span style="border: 1px solid black; padding: 2px;">0</span>	<span style="border: 1px solid black; padding: 2px;">1/2</span>	<span style="border: 1px solid black; padding: 2px;">6</span>	<span style="border: 1px solid black; padding: 2px;">2x</span>

Section B: Solve each equation for x or y.

1.  $7 = 5^x$

$$\ln 7 = \ln 5^x$$

$$\ln 7 = x \ln 5$$

$$\frac{\ln 7}{\ln 5} = x \quad \boxed{x = 1.21}$$

2.  $25e^{x/2} = 750$

$$e^{x/2} = 30$$

$$\ln e^{x/2} = \ln 30$$

$$\ln e = 1 \rightarrow \frac{x}{2} \ln e = \ln 30$$

$$\frac{x}{2} = \ln 30$$

$$x = 2 \cdot \ln 30$$

$$\boxed{x = 6.80}$$

3.  $\log_2 y = 3$

$$2^3 = y$$

$$\boxed{y = 8}$$

4.  $3 \ln x + 2 = 0$

$$3 \ln x = -2$$

$$\ln x = -2/3$$

$$e^{-2/3} = x$$

$$\boxed{x = .513}$$

5.  $\log_2 y + \log_2(y+1) = 1$

$$\log_2 y(y+1) = 1$$

$$2^1 = y(y+1)$$

$$0 = y^2 + y - 2$$

$$(y+2)(y-1) = 0$$

$$y = -2 \quad \boxed{y = 1}$$

6.  $4^y = 32$  (Solve without a calculator)

$$2^{2(y)} = 2^5$$

$$2y = 5$$

$$\boxed{y = 2.5}$$

Section C: Answer the following questions about the equation  $W = 2500(3^{-t/3000})$  where  $W$  is the weight in gram of a radioactive substance after  $t$  years.

1. a) Find the initial weight  
b) Find the weight after 1500 years

a)  $t = 0$  2500 grams

b)  $t = 1500$   
1443.4 grams

2. Find how many years it takes to reduce its value 30%  $\rightarrow$  reduce 30% means 70% remains

initial = 2500  
70% = 1750

$$1750 = 2500(3^{-t/3000})$$

$$.7 = 3^{-t/3000}$$

$$\ln(.7) = -t/3000 \ln(3)$$

$$-.324... = -t/3000$$

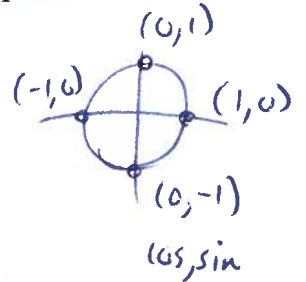
$$t = 973.98$$

$$\boxed{973 \text{ yrs}} \quad 9$$

# IB Math Standard Level Summer Assignment 2017

## TRIGONOMETRIC FUNCTIONS

Section A: Find the exact value of each. ~~There is a blank unit circle at the end of your packet which you may fill in to help you.~~ (Remember: No calculator!)



1.  $\sin 60^\circ$

$\sqrt{3}/2$

2.  $\tan 90^\circ$

$\frac{\sin 90}{\cos 90} = \frac{1}{0} = \text{undef.}$

3.  $\sin \pi$

0

4.  $\tan\left(\frac{\pi}{3}\right)$

$\sqrt{3}$

5.  $\cos\left(\frac{7\pi}{6}\right)$

$-\sqrt{3}/2$



6.  $\cos(-45^\circ)$

$\sqrt{2}/2$



7.  $\tan 135^\circ$

$-1$



8.  $\cos 300^\circ$

$1/2$



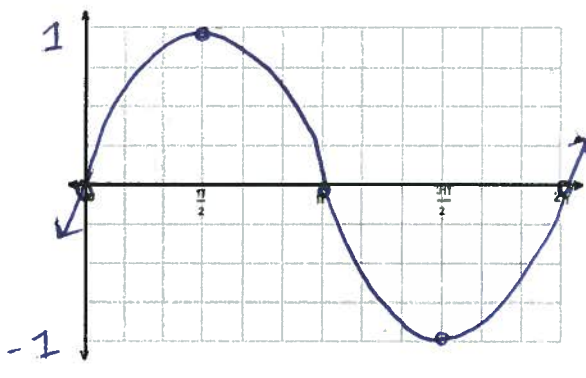
9.  $\sin\left(\frac{4\pi}{3}\right)$

$-\sqrt{3}/2$

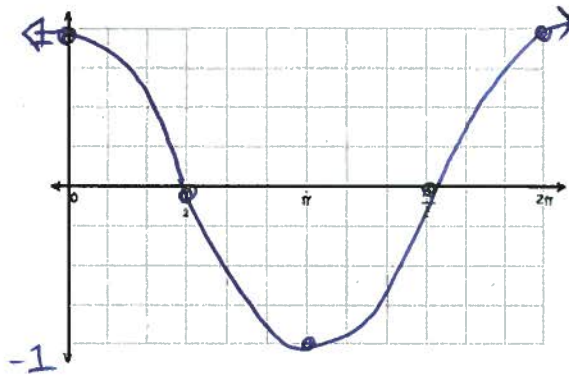


Section B: Graph the functions below on the domain  $0 \leq x \leq 2\pi$  (Remember: No calculator!)

1.  $f(x) = \sin x$



2.  $f(x) = \cos x$



Section C: Solve each trigonometric equation for  $0 \leq x \leq 2\pi$ .

1.  $\sin x = -\frac{1}{2}$



Ref  $\pi/6$   
 $7\pi/6, 11\pi/6$

2.  $2 \cos x = \sqrt{3} \quad \cos x = \sqrt{3}/2$



Ref  $\pi/6$   
 $\pi/6, 11\pi/6$

3.  $4 \sin^2 x = 3$

\*Recall:  $\sin^2 x = (\sin x)^2$

$\sin^2 x = 3/4$

$\sin x = \pm \sqrt{3}/2$  Ref  $\pi/3$

$\pi/3, 2\pi/3, 4\pi/3, 5\pi/3$



4.  $\tan x = 1$



Ref  $\pi/4$   
 $\pi/4, 5\pi/4$