Equations given in Class and on the IB Exam

Topic 1—Algebra

1.1	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
	The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$
	The sum of n terms of a finite geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$
	The sum of an infinite geometric sequence	$S_\infty = \frac{u_1}{1-r}$, $\lvert r \rvert < 1$
1.2	Exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a b$
	Laws of logarithms	$\log_c a + \log_c b = \log_c ab$ $\log_c a - \log_c b = \log_c \frac{a}{b}$ $\log_c a^r = r \log_c a$
	Change of base	$\log_b a = \frac{\log_c a}{\log_c b}$

Topic 2—Functions and equations

2.4	Axis of symmetry of graph of a quadratic function	$f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry $x = -\frac{b}{2a}$
2.6	Relationships between logarithmic and exponential functions	$a^x = e^{x \ln a}$ $\log_a a^x = x = a^{\log_a x}$
2.7	Solutions of a quadratic equation	$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $a \neq 0$
	Discriminant	$\Delta = b^2 - 4ac$

Interval Notation (Not given in class or on the exam... just a refresher)

Description	Interval notation	Description	Interval notation	Description	Interval notation
$x > a$	(a, ∞)	$x \leq a$	$(-\infty, a]$	$a \leq x < b$	$[a, b)$
$x \geq a$	$[a, \infty)$	$a < x < b$	(a, b) - open interval	$a < x \leq b$	$(a, b]$
$x < a$	$(-\infty, a)$	$a \leq x \leq b$	$[a, b]$ - closed interval	All real numbers	$(-\infty, \infty)$

MEMORIZE NOTATION/COMMAND TERMS LIST

NOTATION

<table>
<tr><td>Number Sets</td><td>

$\mathbb{N}$	the set of positive integers and zero, $\{0,1,2,3,...\}$
$\mathbb{Z}$	the set of integers, $\{0,\pm 1,\pm 2,\pm 3,...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1,2,3,...\}$
$\mathbb{Q}$	the set of rational numbers (any # that can be written as a fraction)
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \mid x \in \mathbb{Q}, x > 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \mid x \in \mathbb{R}, x > 0\}$

</td></tr>
<tr><td>Absolute Value</td><td>IB will refer to this as modulus</td></tr>
<tr><td>Line Segments</td><td>Line segment, $\overline{AB}$, will be written as $[AB]$</td></tr>
<tr><td>Angles</td><td>We write Angle A as [redacted] $\angle A$.
IB will use the following notation: $\hat{A}$</td></tr>
<tr><td>Repeating Decimals</td><td>

		IB Notation
$\frac{1}{3}$	$0.\overline{3}$	$0.\dot{3}$
.123123123123	$0.\overline{123}$	$0.\dot{1}2\dot{3}$

</td></tr>
<tr><td>Slope</td><td>IB will refer to this [redacted] the gradient
as</td></tr>
</table>

COMMAND TERMS

Calculate	Obtain a numerical answer showing the relevant stages in the working
Determine	Obtain the only possible answer
Draw	Represent by means of a labeled, accurate diagram or graph, using a pencil. A ruler should be used for straight lines. Diagrams should be drawn to scale. Graphs should have points correctly plotted and joined in a straight line or curve.
Find	Obtain an answer, showing relevant stages in that working.
Hence	Use the proceeding work to obtain the required result
Hence or otherwise	It is suggested that the preceding work is used, but other methods could also receive credit.
Show that	Obtain the required result (possible using information given) without the formalily of proof. "Show that" questions do not generally require the use of a calculator.
Sketch	Represent by means of a diagram or graph (labelled as appropriate) The sketch should give a general idea of the required shape or relationship, and should include relevant features.
Solve	Obtain the answer(s) using algebraic and/or numerical and/or graphical methods.
Write down	Obtain the answer(s), usually by extracting information. Little or no calculation is required. Working does not need to be shown.

Determining the Number of Significant Digits

When counting the significant digits in a number, keep the following rules in mind:

1 All nonzero digits are significant.

Ex: 127 (3 sig digs)

2.5 (2 sig digs)

2 All zeros *between* nonzeros are significant.

Ex: 10204 (5 sig digs)

10.03 (4 sig digs)

3 *Leading* zeros in a decimal are ***not*** significant.

Ex: 0.12 (2 sig digs)

0.034 (2 sig digs)

4 *Trailing* zeros are ***not*** significant *unless* followed by or to the right of a decimal point.

Note: (Actually trailing zeros without any decimal point are ambiguous, their significance should be defined and stated by the person giving the numbers. However most people assume trailing zeros to be insignificant. If the trailing zeros are given as significant the rules for determining the number of significant figures become very simple— go to the first nonzero number and begin counting until you get to the end of the number.)

Using standard assumption that trailing zeros with out a decimal point are insignificant...

Ex: 1540 (3 sig figs)

320 (2 sig figs)

320. (3 sig digs)

320.00 (5 sig digs)

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QUADRATICS

Section A: Factor each quadratic. If the quadratic cannot be factored, write "prime."

1. $x^2 - x - 2$
2. $x^2 + 3x - 4$
3. $8x^2 - 50y^2$
4. $3x^2 - 5x + 2$
5. $2x^2 - x - 6$
6. $x^3 - 3x^2 - 18x$

Section B: Solve each equation using any method except graphing or guess and check.

1. $x^2 + 25 = 10x$
2. $x^2 + 3x - 1 = 0$
3. $x + \frac{12}{x} = 7$
4. $x^2 + 2 = 9$
5. $x^2 - 5x = 0$
6. $36x^2 - 25 = 0$

Section C: State the following for each of the given equations: axis of symmetry, vertex, direction of opening, x-intercepts, and y-intercepts. Then sketch the graph using that information.

1. $y = -2(x+2)(x-1)$
2. $y = 0.5(x-2)^2 - 4$
3. $y = 2x^2 + 6x - 3$

Section D: Find the values of p such that the equations below have the given characteristics. Hint: Use the discriminant.

1. Two different real roots
   $px^2 + 5x + 2 = 0$
2. Two equal real roots
   $2x^2 - 3x + p = 0$
3. No real roots
   $px^2 - 4px + 5 - p = 0$

Section E: Use your graphing calculator to find the following

1. Solve $3x^2 - x - 5 = 0$
2. Intersection points of $y = -x^2 - 5x + 3$ and $y = x^2 + 3x + 11$

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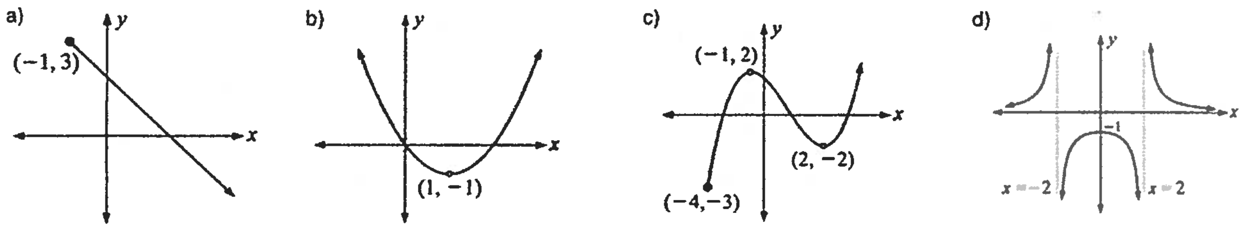
FUNCTIONS

Section A: For each of the following find the domain and range without using a calculator.

1.



2.

a) $f(x) = \sqrt{x}$ b) $f(x) = \sqrt{4 - x}$ c) $y = 5x - 3x^2$ d) $y = \frac{x+4}{x-2}$

Section B: Find the inverse of each function.

1. $f(x) = 2x + 1$
2. $f(x) = \frac{x^2}{3}$
3. $g(x) = \frac{5}{x-2}$
4. $g(x) = \sqrt{4 - x} + 1$
5. If the point (2, 7) is on the graph of $f(x)$, what point must be on the graph of $f^{-1}(x)$?
6. Explain, in complete sentences, the relationship between a function and its inverse.

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Section C: Let $f(x) = 2x^2 - 1$; $g(x) = 3x$ and $h(x) = 5 - x$. Find the following.

1. $f(-3)$
2. $(f \circ g)(x)$
3. $(h \circ f)(x)$
4. $(f \circ h)(x + 1)$
5. $(g \circ h)(4)$
6. $(f \circ f)(-1)$

Section D: Answer the following questions concerning equations of lines

1. What is the slope, x-intercept, and y-intercept of the equation $5x - 4y = 8$?
2. What is the slope-intercept form of the equation of the line between the points (4, 3) and (7, –2) ?
3. What is the slope-intercept form of a line perpendicular to $y = -2x + 9$ passing through the (4, 7) ?

Section E: Find the horizontal & vertical asymptotes and holes (if applicable) of the following.

1. $y = \frac{1}{2x-5}$
2. $y = \frac{x^2-5}{2x^2-12}$
3. $y = \frac{x^2+2x-3}{x^3+6x^2-7x}$

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Section F: For each pair of functions $f(x)$ and $g(x)$, describe the transformations that would transform $f(x)$ into $g(x)$.

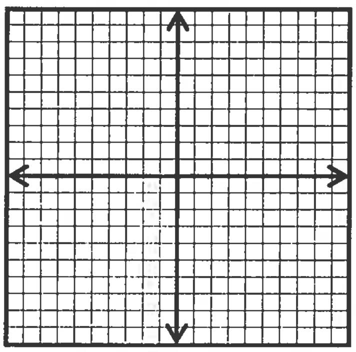
1. $f(x) = x^2$; $g(x) = (x-5)^2 + 2$

2. $f(x) = \sqrt{x}$; $g(x) = \sqrt{3x} - 10$

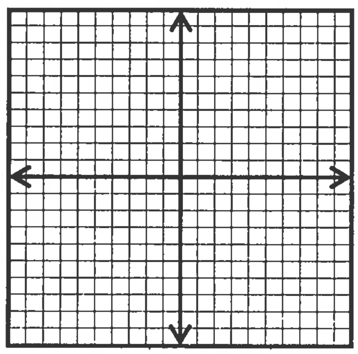
3. $f(x) = e^x$; $g(x) = -5(e)^{x-1}$

Section G: Graph each function, clearly showing its key features (maxima, minima, and intercepts). Identify its domain and range. (Remember: No calculator!)

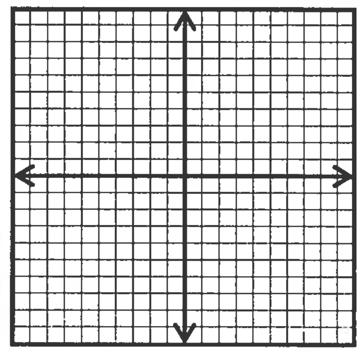
1. $f(x) = x^2 - 5$



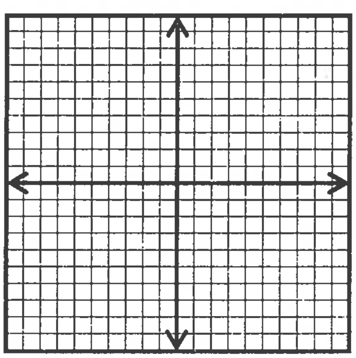
2. $f(x) = 3x - 4$



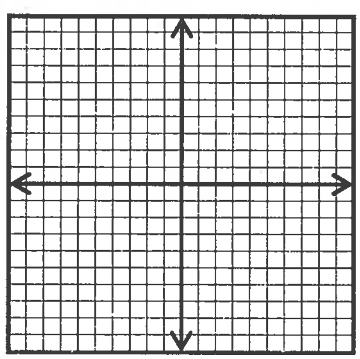
3. $f(x) = x^3 + 1$



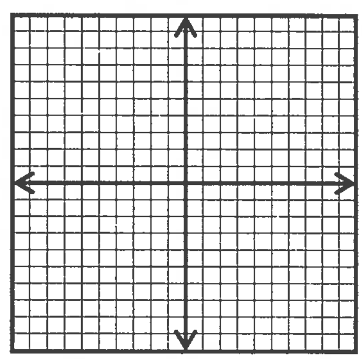
4. $f(x) = \sqrt{x+6}$



5. $f(x) = |x-1| + 3$



6. $f(x) = 2^x - 4$



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ALGEBRA

Section A: Simplify the following without a calculator.

1. $(2x^5)^{-3}$
2. $8^{2/3}$
3. $81^{-3/4}$
4. $\sqrt[3]{16x^3}$
5. $\sqrt{10x^2}\cdot\sqrt{70x^6}$
6. $\frac{\sqrt{72x^4}}{\sqrt{3x}}$
7. $\frac{5}{7-\sqrt{5}}$
8. $\sqrt{5}-5\sqrt{125}-7\sqrt{180}$

Section B: Solve using algebra.

1. $3x+7y=36$, $x=5y-10$
2. $6x+10y=32$, $4x-2y=4$
3. $x=y^2$, $x-y=6$
4. $x^2+y^2=25$, $y=x^2-13$

Section C: Solve for x. Eliminate any extraneous solutions, if any.

1. $\sqrt{37-3x}=x-3$
2. $-3(2x+1)^3=-192$
3. $\frac{x}{3}-\frac{5}{2}=\frac{-3}{x}$
4. $\frac{4x-1}{x+1}=x-1$
5. $2|3x-1|+5=-2x+8$
6. $5(x-3)\le 8(x+5)$

SEQUENCES AND SERIES

Section A: Answer the following questions concerning arithmetic sequences and series.

1. Consider the sequence 87, 83, 79, 75…

a) What is the formula for the general term u_n?

b) What is the 40^{th} term?

c) Is –143 a member?

d) What is the sum of the first 22 terms?

2. A sequence is defined by $u_n = 3n - 2$

a) What is u_1 and d?

b) What is the 57^{th} term?

c) What is the first term to exceed 450?

d) What is the sum of the first 57 terms?

3) Find the general term u_n for an arithmetic sequence given that $u_7 = 41$ and $u_{13} = 77$.

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Section B: Answer the following questions concerning arithmetic sequences and series.

1. Consider the sequence 12, –6, 3, –1.5, …

a) What is the formula for the general term u_n?

b) What is the 13^{th} term?

c) What is the sum of the first 10 terms?

d) What is the infinite sum of the sequence‘

2. Find the general term u_n for an geometric sequence given that $u_4 = 24$ and $u_7 = 192$.

3. In 1998 there were 3000 koalas on Koala Island. Since then, the population of koalas on the island has increased by 5% each year. How many koalas were on the island in 2001? In what year will the population first exceed 5000?

Section C: Find the following sums written in Sigma Notation.

1. $\sum_{r=1}^{4}(3r-5)$

2. $\sum_{i=1}^{15} 50(0.8)^{i-1}$

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EXPONENTIAL AND LOGARITHMIC EQUATIONS

Section A: Find the following without using a calculator.

1.

a) $\log_4 64$ b) $\log_2 1/4$ c) $\log_8 1$ d) $\log_9 3$ e) $\log_m m^6$ f) $\ln(e^{2x})$

Section B: Solve each equation for x or y.

1. $7 = 5^x$
2. $25e^{x/2} = 750$
3. $\log_2 y = 3$
4. $3\ln x + 2 = 0$
5. $\log_2 y + \log_2(y + 1) = 1$
6. $4^y = 32$ (Solve without a calculator)

Section C: Answer the following questions about the equation $W = 2500\left(3^{-t/3000}\right)$ where W is the weight in gram of a radioactive substance after t years.

1. a) Find the initial weight
   b) Find the weight after 1500 years
2. Find how many years it takes to reduce its value 30%

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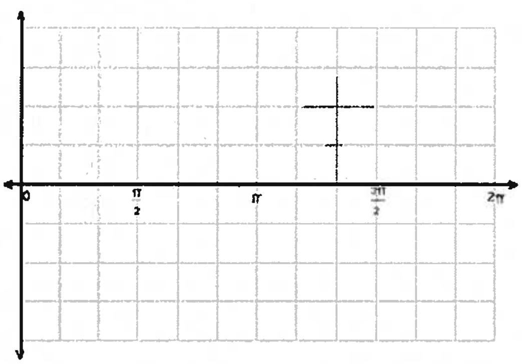
TRIGONOMETRIC FUNCTIONS

Section A: Find the exact value of each. Remember: No calculator!

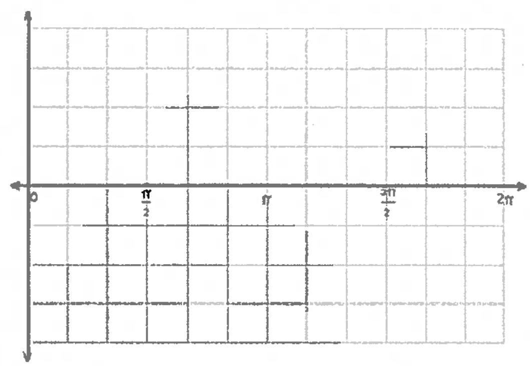
1. $\sin 60°$
2. $\tan 90°$
3. $\sin \pi$
4. $\tan\left(\frac{\pi}{3}\right)$
5. $\cos\left(\frac{7\pi}{6}\right)$
6. $\cos(-45°)$
7. $\tan 135°$
8. $\cos 300°$
9. $\sin\left(\frac{4\pi}{3}\right)$

Section B: Graph the functions below on the domain $0 \leq x \leq 2\pi$ (Remember: No calculator!)

1. $f(x) = \sin x$



2. $f(x) = \cos x$



Section C: Solve each trigonometric equation for $0 \leq x \leq 2\pi$.

1. $\sin x = -\frac{1}{2}$
2. $2\cos x = \sqrt{3}$
3. $4\sin^2 x = 3$
   *Recall: $\sin^2 x = (\sin x)^2$
4. $\tan x = 1$