c $QPR = 180^0 - 100^0 - 30.5^0$

$QPR = 49.5^0$

$RPM = 90^0 - 49.5^0$

$RPM = 40.5^0$

$\sin RPM = \frac{7.8+h}{PR}$

$\sin 40.5° = \frac{7.8+h}{15.9235...}$

$10.34... = 7.8 + h$

$h = 2.54$ m (3 s.f.)

Ch. 3&6

Exercise 3P

1 Use the area of a triangle formula.

a $A = \frac{1}{2} \times 12 \times 7 \times \sin 82°$

$A = 41.6$ km^2 (3 s.f.)

b $A = \frac{1}{2} \times 81.7 \times 60.5 \times \sin 50°$

$A = 1890$ m^2 (3 s.f.)

2 a ABC is an isosceles triangle

$B = 180° - 2 \times 40°$

$B = 100°$

b $A = \frac{1}{2} \times 10 \times 10 \times \sin 100°$

$A = 49.2$ cm^2

3 a $C = 180° - 2 \times 50°$

$C = 80°$

b $A = \frac{1}{2} \times 3 \times 3 \times \sin 80$

$A = 4.43$ m^2

4 Find first the size of one angle.

$\cos X = \frac{20^2 + 16^2 - 8^2}{2 \times 20 \times 16}$

$\cos X = 0.925$

$X = \cos^{-1}(0.925)$

$X = 22.331...°$

$A = \frac{1}{2} \times 20 \times 16 \times \sin 22.3316...°$

$A = 60.8$ km^2 (3 s.f.)

5 a $\frac{10}{\sin 100°} = \frac{5}{\sin Y}$

$\sin Y = \frac{5 \sin 100°}{10}$

$\sin Y = 0.4924...$

$Y = \sin^{-1}(0.4924...)$

$Y = 29.498....°$

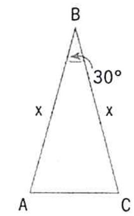
$Z = 180° - 100° - 29.4987...$

$Z = 50.5°$ (3 s.f.)

b $A = \frac{1}{2} \times 50 \times 100 \times \sin 50.5012...°$

$A = 1930\,\text{m}^2$ (nearest $10\,\text{m}^2$)

6 a



$4 = \frac{1}{2} \times x \times x \times \sin 30°$

$4 = \frac{1}{2} \times x^2 \times 0.5$

$4 = 0.25 \times x^2$ or equivalent

b $4 = 0.25 \times x^2$

$x^2 = 16$

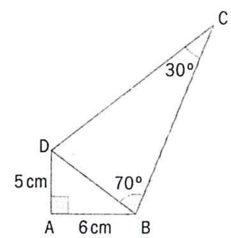
$x = 4$ cm

7 a ABD is a right-angled triangle.

$DB^2 = 5^2 + 6^2$

$DB = \sqrt{61}$ cm or 7.81 cm (3 s.f.)

b in triangle BCD



$\frac{\sqrt{61}}{\sin 30°} = \frac{DC}{\sin 70°}$

$DC = \frac{\sqrt{61} \sin 70°}{\sin 30°}$

$DC = 14.7$ cm (3 s.f.)

c from parts **a** and **b**.

$BDC = 80^0$

$A = \frac{1}{2} \times \sqrt{61} \times 14.678.. \times \sin 80°$

$A = 56.5$ cm^2 (3 s.f.)

d Area of ABCD = Area of ABD + Area of BCD

Area of ABCD = $\frac{1}{2} \times 6 \times 5 + 56.450...$

Area of ABCD = 71.5 cm^2 (3 s.f.)

Review exercise

Paper 1 style questions

1 a $A(1, 3)$ and $B(5, 1)$

$m = \frac{1-3}{5-1}$

$m = -\frac{1}{2}$

b parallel lines have the same gradient. $y = -\frac{1}{2}x + c$

L_2 passes through $(0, 4)$

$y = -\frac{1}{2}x + 4$ or equivalent forms.

2 a use the gradient formula

$A(0, 6)$ and $B(6, 0)$

$m = \frac{0-6}{6-0}$

$m = -1$

b perpendicular lines have gradients that are opposite and reciprocal.

$m_{\perp} \times m = -1$

$m_{\perp} = \frac{-1}{m}$

$m_{\perp} = \frac{-1}{-1}$

$m_{\perp} = 1$

c $y = 1x + c$

L_2 passes through O (0, 0)

$y = 1x + 0$

$y = x$

3 a i A line meets the x-axis at the point where

$y = 0$

$y = 2x + 3$

$0 = 2x + 3$

$x = -\frac{3}{2}$ (or -1.5)

Point is $\left(\frac{-3}{2}, 0\right)$

ii A line meets the y-axis at the point where

$x = 0$

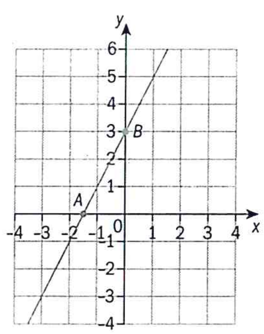
$y = 2x + 3$

$y = 2 \times 0 + 3$

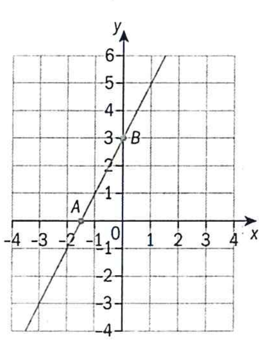
$y = 3$

Point is (0, 3)

b Use the two points found in **a** and draw the line.



c



$\tan\alpha = \frac{3}{1.5}$

$\tan\alpha = 2$

$\alpha = \tan^{-1} 2$

$\alpha = 63.4°$ (3 s.f.)

4 If a point lies on a line then its coordinates verify the equation of the line.

a $y = -2x + 6$

$(a, 4)$ lies on L_1

$4 = -2a + 6$

$a = 1$

b $y = -2x + 6$

$(12.5, b)$ lies on L_1

$b = -2 \times 12.5 + 6$

$b = -19$

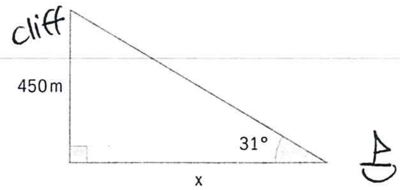
c use the GDC.

$3x - y + 1 = 0$

$y = 3x + 1$ and $y = -2x + 6$

The point is (1, 4)

5 a



b $\tan 31° = \frac{450}{x}$

$x = \frac{450}{\tan 31°}$

$x = 749$ m (3 s.f.)

6 a Sum of the interior angles of a triangle is 180°.

$2 \times 32° + CAB = 180°$

$CAB = 116°$

b $\frac{AB}{\sin 32°} = \frac{20}{\sin 116°}$

$AB = \frac{20 \sin 32°}{\sin 116°}$

$AB = 11.8$ cm (3 s.f.)

c $A = \frac{1}{2} \times 20 \times 11.791\ldots \times \sin 32°$

$A = 62.5$ cm^2 (3 s.f.)

7 a AC = 20 − 5 − 6

= 9 m

b Using the cosine rule,

$\cos BAC = \frac{5^2 + 9^2 - 6^2}{2 \times 5 \times 9}$

$BAC = \cos^{-1}(0.777\ldots)$

$BAC = 38.9°$ (3 s.f.)

c $A = \frac{1}{2} \times 5 \times 9 \times \sin 38.9$

$= 14.1$ m^2

8 a AO = OB = $\frac{10}{2}$ = 5 cm

$\cos AOB = \frac{5^2 + 5^2 - 7.5^2}{2 \times 5 \times 5}$

$\cos AOB = -0.125$

$AOB = \cos^{-1}(-0.125)$

$AOB = 97.2°$

b $A = \frac{1}{2} \times 5 \times 5 \times \sin 97.180\ldots°$

$A = 12.4$ cm^2 (3 s.f.)

c Shaded area = $\pi 5^2 - 12.401\ldots$

Shaded area = 66.1 cm^2 (3 s.f.)

Paper 2 style Questions

1) $5x-7y-8=0$
$5x-8=7y$
$5/7x-8/7\ y$

$3x+ky+11=0$
$ky=-3x-11$
$y=-3/k\,x-11/k$

parallel:
$5/7=-3/k$
$5k=-21$
$k=-21/5$ or -4.2

perpendicular:
$-7/5=-3/k$
$-7k=-15$
$k=15/7$

2) PQ:
$\sqrt{(5-1)^2+(7-5)^2}$
$\sqrt{4^2+2^2}=\sqrt{16+4}=\sqrt{20}$

QR:
$\sqrt{(3-5)^2+(1-7)^2}$
$\sqrt{(-2)^2+(-6)^2}=\sqrt{4+36}=\sqrt{40}$

PR
$\sqrt{(3-1)^2+(1-5)^2}$
$\sqrt{2^2+(-4)^2}=\sqrt{4+16}=\sqrt{20}$

a Since PQ = PR
The triangle is isosceles.

b midpoint QR:
$$\left(\frac{5+3}{2},\frac{7+1}{2}\right)=\left(\frac{8}{2},\frac{8}{2}\right)=(4,4)=M$$

c Slope PM: (1,5)(4,4)
$$\frac{4-5}{4-1}=\frac{-1}{3}$$

Slope QR: (5,7)(3,1)
$$\frac{1-7}{3-5}=\frac{-6}{-2}=3$$

Since the slopes are opposite reciprocals (or have a product of −1), they are perpendicular

3) let s = small cans and l = large cans

$$\begin{aligned} 12s + 16l &= 9 \\ 4s + 12l &= 6 \end{aligned} \Rightarrow \begin{aligned} 12s + 16l &= 9 \\ -3(4s + 12l &= 6) \end{aligned}$$

$$\begin{aligned} 12s + 16l &= 9 \\ -12s - 36l &= -18 \\ \hline -20l &= -9 \\ l &= .45 \end{aligned} \qquad s = .15$$

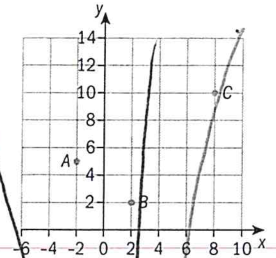
large can = .45 litres
small can = .15 litres

See next page for the rest

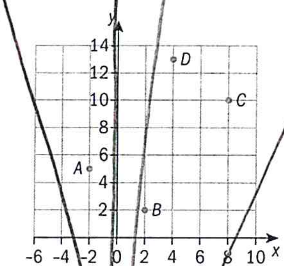
Review exercise

Paper 2 style questions

~~1~~ **a**



b **i**



ii D(4, 13)

c B(2, 2) and C(8, 10)

$m = \frac{10-2}{8-2}$

$m = \frac{8}{6}$ or $\frac{4}{3}$

d DC and BC are perpendicular lines.

$m_{\perp} \times m = -1$

$m_{\perp} \times \frac{4}{3} = -1$

$m_{\perp} = -\frac{3}{4}$

e use the gradient formula $-\frac{3}{4} = \frac{y-10}{x-8}$

$3(x - 8) = -4(y - 10)$

$3x + 4y - 64 = 0$

f **i** C(8, 10) and D(4, 13)

$d = \sqrt{(4-8)^2 + (13-10)^2}$

$d = 5$

ii B(2, 2) and C(8, 10)

$d = \sqrt{(8-2)^2 + (10-2)^2}$

$d = 10$

g $\tan DBC = \frac{5}{10}$

$DBC = \tan^{-1}\left(\frac{5}{10}\right)$

$DBC = 26.6°$ (3 s.f.)

4 **a** Let x be the length of the ladder.

$\cos 60° = \frac{2}{x}$

$x = \frac{2}{\cos 60°}$

$x = 4$ m

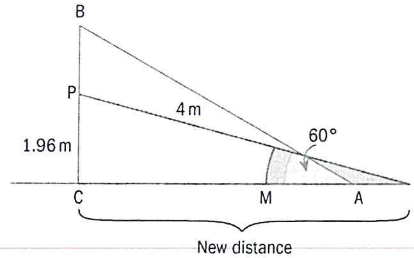
b Let y be the height of the pole.

$\tan 60° = \frac{y}{2}$

$y = 3.46$ m (3 s.f.)

c $3.4641... - 1.5 = 1.96$ m (3 s.f.)

d The length of the ladder is still the same.



Let the new distance be d

$d^2 + 1.9641...^2 = 4^2$

$d = 3.48$ (3 s.f.)

e $\tan\beta = \frac{1.9641...}{3.4845...}$

$\beta = \tan^{-3}\left(\frac{1.9641...}{3.4845...}\right)$

$\beta = 29.4°$ (3 s.f.)

5 **a** in triangle BCD, $BD^2 = 300^2 + 400^2$

$BD = 500$ m

b in triangle BCD, $\tan BDC = \frac{300}{400}$

$BDC = \tan^{-1}\left(\frac{300}{400}\right)$

$BDC = 36.87°$ (2 d.p.)

c angle ADC = 108°

ADB = 108° – 36.87° = 71.1° (3 s.f.)

d In triangle ADB.

$AB^2 = 500^2 + 1200^2 - 2 \times 500 \times 1200 \times \cos 71.1°$

$AB = 1140$ m (3 s.f.)

e **i** Perimeter = 1200 + 400 + 300 + 1141.00...

Perimeter = 3040 m (3 s.f.)

ii rate ~~velocity~~ $= \frac{\text{distance}}{\text{time}}$

$3.8 = \frac{3040}{\text{time}}$

$\text{time} = \frac{3040}{3.8}$

time = 800 seconds

$\text{time} = \frac{800}{60}$ minutes = 13 minutes (nearest minute)

f split the quadrilateral in two triangles

Area ABCD = Area ADB + Area BDC

Area ABCD =

$\frac{1}{2} \times 1200 \times 500 \times \sin 71.1° + \frac{1}{2} \times 400 \times 300$

Area ABCD = 343825 m² = 343825×10^{-6} km² = 0.344 km².