c $AOB = \frac{360°}{6}$

$AOB = 60°$

d Let x be the length of AB.

Area of AOB = 6 cm^2

Area of AOB $= \frac{1}{2} \times x \times x \times \sin 60^{\circ}$

Therefore

$6 = \frac{1}{2} \times x \times x \times \sin 60^{\circ}$

$6 = \frac{1}{2} x^2 \times \sin 60^{\circ}$

$6 = \frac{1}{2} x^2 \times \sin 60^{\circ}$

$x = 3.72$

AB = 3.72 cm (3 s.f.)

Section 6.6 & 6.7

5 a Volume of sphere $= \frac{4}{3}\pi r^3$

$200 = \frac{4}{3}\pi r^3$

$\frac{200 \times \frac{3}{4}}{\pi} = r^3$

$r = \sqrt[3]{\frac{200 \times \frac{3}{4}}{\pi}}$

$r = 3.63$ cm (3 s.f.)

b $r = 3.63$ cm $= 3.63 \times 10$ mm $= 36.3$ mm

36.3 mm = 36 mm correct to the nearest millimetre.

6 a Volume of cylinder $= \pi r^2 h$

Volume of cylinder $= \pi \times 15^2 \times 30$

Volume of cylinder $= (6750)\pi$ cm^3 or

21200 cm^3 (3 s.f.)

b Volume of cuboid $= l \times w \times h$

Volume of cuboid $= 60 \times 20 \times 17$

Volume of cuboid = 20400 cm^3

There is not enough space as 21200 > 20400.

Review exercise

Paper 1 style questions

1 a Surface area of ABCDEFGH =
$2 \times (20 \times 42) + 2 \times (20 \times 34) + 2 \times (34 \times 42)$

Surface area of ABCDEFGH = 5896 cm^2

in dm

b Volume of cuboid $= l \times w \times h$

Volume of cuboid $= 34 \times 42 \times 20$

Volume of cuboid = 28560 cm^3

28560 cm^3 $= 28560 \times 10^{-3}$ dm^3 $= 28.56$ dm^3

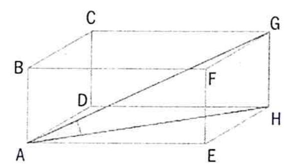
2 a AH is the hypotenuse of a triangle.

$AH^2 = AE^2 + EH^2$

$AH^2 = 10^2 + 4^2$

$AH = \sqrt{116}$ cm or 10.8 cm (3 s.f.)

b



$\tan HAG = \frac{5}{\sqrt{116}}$

$HAG = \tan^{-1}\left(\frac{5}{\sqrt{116}}\right)$

$HAG = 24.9°$ (3 s.f.)

3 a AC is the diagonal of the base.

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 5^2$

$AC = \sqrt{41}$ cm or 6.40 cm (3 s.f.)

b $EC^2 = EO^2 + OC^2$

$EC^2 = EO^2 + \left(\frac{AC}{2}\right)^2$

$EC^2 = 8^2 + \left(\frac{\sqrt{41}}{2}\right)^2$

$EC = \sqrt{74.25}$ cm or 8.62 cm (3 s.f.)

c AEC is an isosceles triangle.

$\cos AEC = \frac{AE^2 + EC^2 - AC^2}{2 \times AE \times EC}$

$\cos AEC = \frac{(\sqrt{74.25})^2 + (\sqrt{74.25})^2 - (\sqrt{41})^2}{2 \times \sqrt{74.25} \times \sqrt{74.25}}$

$AEC = \cos^{-1}\left(\frac{(\sqrt{74.25})^2 + (\sqrt{74.25})^2 - (\sqrt{41})^2}{2 \times \sqrt{74.25} \times \sqrt{74.25}}\right)$

$AEC = 43.6°$ (3 s.f.)

4 a Let the midpoint be M

$EO^2 + OM^2 = EM^2$

$9^2 + 3^2 = EC^2$

$EC = \sqrt{90}$ cm or 9.49 cm (3 s.f.)

b Area of triangle BCE $= \frac{1}{2} \times 6 \times \sqrt{90}$

Area of triangle BCE = 28.5 cm^2 (3 s.f.)

c Surface area of pyramid
= 4 × area of triangle BEC + Area of base
Surface area of pyramid = 4 × 28.46....+ 6^2
Surface area of pyramid = 150 cm^2 (3 s.f.)

5 **a** Let x be the edge length of the cube.

Volume of cube $= x^3$

$512 = x^3$

$\sqrt[3]{512} = x$

$x = 8$ cm

b AC is the diagonal of the base.

$AC^2 = AB^2 + BC^2$

$AC^2 = 8^2 + 8^2$

$AC^2 = 128$

$AC = \sqrt{128}$ cm or 11.3 cm (3 s.f.)

c $AG^2 = AC^2 + CG^2$

$AG^2 = \left(\sqrt{128}\right)^2 + 8^2$

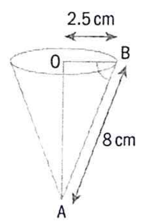
$AG^2 = 192$

$AG = \sqrt{192}$ cm or 13.9 cm (3 s.f.)

13.5 < 13.9, therefore the pencil fits in the cube.

6 Triangle AOB is a right-angled triangle.

a



$\cos OBA = \frac{2.5}{8}$

$OBA = \cos^{-1}\left(\frac{2.5}{8}\right)$

$OBA = 71.8°$ (3 s.f.)

b **i** $AB^2 = AO^2 + OB^2$

$8^2 = AO^2 + 2.5^2$

$AO^2 = 8^2 - 2.5^2$

$AO = \sqrt{57.75}$ cm or 7.60 cm (3 s.f.)

ii Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone $= \frac{1}{3}\pi \times 2.5^2 \times \sqrt{57.75}$

Volume of cone $= 49.7$ cm^3 (3 s.f.)

7 **a** Area of triangle ABC $= \frac{1}{2} \times 2.4 \times 2.4 \times \sin 110°$

Area of triangle ABC $= 2.71$ m^2 (3 s.f.)

b Volume = area of cross section × height

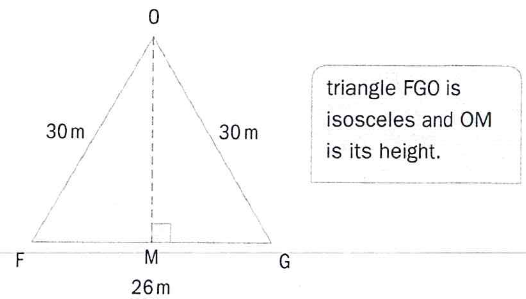
Volume = 2.706... × 3.5

Volume = 9.47m^3 (3 s.f.)

Review exercise

Paper 2 style questions

1 **a**



$OM^2 + MG^2 = GO^2$

$OM^2 + 13^2 = 30^2$

$OM^2 = 30^2 - 13^2$

$OM = \sqrt{731}$ m or 27.0 m (3 s.f.)

b The height of the tower is the addition of the height of the pyramid and the height of the cuboid.

Let P be the midpoint of the base of the pyramid.

$OP^2 + PM^2 = OM^2$

$OP^2 + 13^2 = \left(\sqrt{731}\right)^2$

$OP^2 = \left(\sqrt{731}\right)^2 - 13^2$

$OP^2 = 562$

$OP = \sqrt{562}$

Height of the tower = OP + height of cuboid

Height of the tower = $\sqrt{562}$ + 70

Height of the tower = 93.7 m (3 s.f.)

c $\cos OMP = \frac{13}{\sqrt{731}}$

$OMP = \cos^{-1}\left(\frac{13}{\sqrt{731}}\right)$

$OMP = 61.3°$ (3 s.f.)

d Surface area $= 4 \times (26 \times 70) + 4 \times \left(\frac{1}{2} \times 26 \times \sqrt{731}\right)$

Surface area = 8685.9246...m^2

Cost of cleaning = 78 × 8685.9246... m^2

Cost of cleaning = USD 677502 (correct to the nearest dollar)

2 **a**

$$\text{Volume of hemisphere} = \frac{\frac{4}{3}\pi r^3}{2}$$

$$\text{Volume of hemisphere} = \frac{\frac{4}{3}\pi \times 3^3}{2}$$

$$\text{Volume of hemisphere} = \frac{\frac{4}{3}\times 3^3}{2}\pi$$

Volume of hemisphere = (18π) cm^2

b Volume of cone $=\frac{1}{3}\times\pi\times 3^2\times h$

Volume of hemisphere = (18π) cm^3

Therefore

$$\frac{2}{3}\left(\frac{1}{3}\times\pi\times 3^2\times h\right)=18\pi$$

$$\frac{1}{3}\times\pi\times 3^2\times h=\frac{18\pi}{\frac{2}{3}}$$

$$\pi\times 3\times h = 27\pi$$

$$h=\frac{27\pi}{\pi\times 3}$$

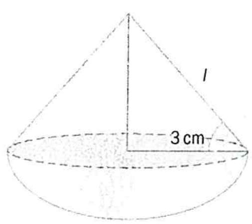
$h = 9$ cm

c $l^2 = 3^2 + 9^2$

$l^2 = 90$

$l=\sqrt{90}$ cm or 9.49 cm (3 s.f.)

d



Let the angle be α.

$$\tan\alpha=\frac{9}{3}$$

$$\alpha=\tan^{-1}\left(\frac{9}{3}\right)$$

$\alpha = 71.6°$ (3 s.f.)

e Volume of sculpture = Volume of hemisphere + Volume of cone

Volume of sculpture $= 18\pi + \frac{1}{3}\times\pi\times 3^2\times 9$

Volume of sculpture $= 18\pi + 27\pi$

Volume of sculpture = (45π) cm^3

Weight of sculpture $= 45\pi\times 10.8$

Weight of sculpture = 1530 grams (3 s.f.)

Therefore

1530 grams = 1.53 kg

3 **a** Volume of pyramid $=\frac{1}{3}$(area of base × height)

Volume of pyramid $=\frac{1}{3}(5^2\times 7)$

Volume of pyramid $=\frac{175}{3}$ cm^3 or 58.3 cm^3 (3 s.f.)

b Weight of the pyramid $=\frac{175}{3}\times 8.7 = 507.5$ grams

507.5 grams = 508 grams (correct to the nearest grams)

c EB is the hypotenuse of EOB.

$DB^2 = DA^2 + AB^2$

$DB^2 = 5^2 + 5^2$

$DB=\sqrt{50}$

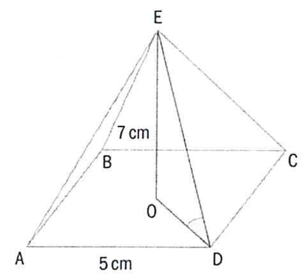
Now we find EB

$EB^2 = EO^2 + OB^2$

$$EB^2 = 7^2+\left(\frac{\sqrt{50}}{2}\right)^2$$

$EB=\sqrt{61.5} = 7.842$ cm (4 s.f.)

d

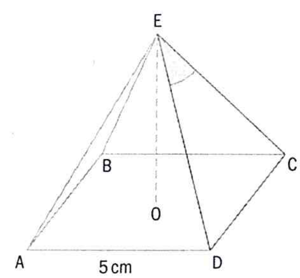


$$\sin ODE=\frac{7}{7.842}$$

$$ODE=\sin^{-1}\left(\frac{7}{7.842}\right)$$

$ODE=63.2°$ (3 s.f.)

e



$$\cos DEC=\frac{DE^2+EC^2-CD^2}{2\times DE\times EC}$$

$$\cos DEC=\frac{7.842^2+7.842^2-5^2}{2\times 7.842\times 7.842}$$

$$DEC=\cos^{-1}\left(\frac{7.842^2+7.842^2-5^2}{2\times 7.842\times 7.842}\right)$$

$DEC=37.2°$ (3 s.f.)

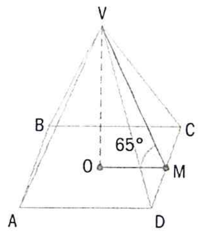
f Surface area = 4 × Area of DEC + Area of base

Surface area

$$=4\times\left(\frac{1}{2}\times 7.842\times 7.842\right)\times\sin 37.18° + 5^2$$

Surface area = 99.3cm^2 (3 s.f.)

4 a



$$\tan 65° = \frac{VO}{4}$$

$VO = 4 \tan 65°$

$VO = 8.58$ cm (3 s.f.)

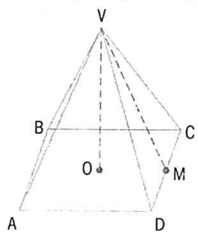
b i in triangle VMO

$$\cos 65° = \frac{4}{VM}$$

$$VM = \frac{4}{\cos 65°}$$

$VM = 9.46$ cm (3 s.f.)

ii $\angle DVC = 2 \times \angle MVC$



$$\tan MVC = \frac{CM}{MV}$$

$$\tan MVC = \frac{4}{9.46}$$

$$MVC = \tan^{-1}\left(\frac{4}{9.46}\right)$$

$MVC = 22.92....$

$DVC = 2 \times MVC$

$DVC = 2 \times 22.92...$

$DVC = 45.8°$

c Surface area of the pyramid = 4 × Area of DVC + Area of base

Surface area of the pyramid

$$= 4 \times \left(\frac{1}{2} \times 9.46 \times 8\right) + 8^2$$

Surface area of the pyramid = 215cm²

d Volume of pyramid $= \frac{1}{3}$(Area of base × Height)

Volume of pyramid $= \frac{1}{3}(8^2 \times 8.58)$

Volume of pyramid = 183cm³

e Volume of cone = volume of pyramid

$V = 183\text{ cm}^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$r = 4$ (½ of 8)

$183 = \frac{1}{3}\pi(4)^2 h$

$549 = \pi \cdot 16 h$

$$\frac{549}{\pi \cdot 16} = h$$

$h = 10.9$ cm

(3 sig figs)