Differentiate each function with respect to x.

1) $f(x) = 5x^4$

$$f'(x) = 20x^3$$

2) $f(x) = -4x^3$

$$f'(x) = -12x^2$$

3) $f(x) = -4x$

$$f'(x) = -4$$

4) $f(x) = 3$

$$f'(x) = 0$$

5) $y = -4x^4 + 5x^3 + 2x$

$$y' = -16x^3 + 15x^2 + 2$$

$$\text{or } \frac{dy}{dx} = -16x^3 + 15x^2 + 2$$

6) $y = 3x^5 + 4x + 5$

$$y' = 15x^4 + 4$$

7) $y = \frac{4}{3}x^3 + 2x$

$$y' = 4x^2 + 2$$

8) $f(x) = \frac{5}{3}x^4 + \frac{1}{5}x^2$

$$f'(x) = \frac{20}{3}x^3 + \frac{2}{5}x$$

9) $y = 4x + 3x^{-5}$

$$y' = 4 - 15x^{-6}$$

$$y' = 4 - \frac{15}{x^6}$$

10) $f(x) = 4x^4 + \frac{5}{x^3} = 4x^4 + 5x^{-3}$

$$f'(x) = 16x^3 - 15x^{-4}$$

$$= 16x^3 - \frac{15}{x^4}$$

11) $y = 4x^{-2} + \frac{5}{x^3} = 4x^{-2} + 5x^{-3}$

$$y' = -8x^{-3} - 15x^{-4}$$

$$y' = \frac{-8}{x^3} - \frac{15}{x^4}$$

12) $f(x) = \frac{1}{x^2} + \frac{4}{x^4} = x^{-2} + 4x^{-4}$

$$f'(x) = -2x^{-3} - 16x^{-5}$$

$$= \frac{-2}{x^3} - \frac{16}{x^5}$$

For each problem, find $f'(1)$.

13) $f(x) = -x^3 + 4x^2 - 6$

$f'(x) = -3x^2 + 8x$

$f'(1) = -3(1)^2 + 8(1) = -3(1) + 8 = -3 + 8 = \boxed{5}$

For each problem, find the gradient (slope) of the function at the given value.

14) $y = -x^2 + 4x$ at $x = 3$

$y' = -2x + 4$

$= -2(3) + 4 = -6 + 4 = \boxed{-2}$

For each problem, find the equation of the line tangent to the function at the given value. Your answer should be in slope-intercept form.

15) $y = x^2 + 8x + 13$ at $x = -2$

first find y...

$(-2)^2 + 8(-2) + 13 = 1$

pt $(-2, 1)$

$y' = 2x + 8$

plug in $x = -2$ to find slope

$2(-2) + 8 = 4$

$m = 4$

Tangent Line: $m = 4$ pt $(-2, 1)$

$y - 1 = 4(x + 2)$

$y - 1 = 4x + 8$

$\boxed{y = 4x + 9}$

For each problem, find the equation of the line normal to the function at the given point. Your answer should be in slope-intecept form.

16) $y = \dfrac{x^2}{2} - 4x + 5$ at $x = 0$

first find y...

$y = \dfrac{0^2}{2} - 4(0) + 5$

pt $(0, 5)$

$y = \frac{1}{2}x^2 - 4x + 5$

$y' = 1x - 4$

plug in $x = 0$

$0 - 4 = -4$ $m = -4$

Normal Line: $m = \frac{1}{4}$ pt $(0, 5)$

$y - 5 = \frac{1}{4}(x - 0)$

$y - 5 = \frac{1}{4}x$

$\boxed{y = \frac{1}{4}x + 5}$

For each problem, find the points where the tangent line to the function is horizontal.

17) $y = -x^3 + 3x^2 - 6$

first find y' then set equal to 0

$y' = -3x^2 + 6x$

$0 = -3x^2 + 6x$

$0 = -3x(x - 2)$

$x = 0 \quad x = 2$

plug them back into the original to find the points

$x = 0: -(0)^3 + 3(0)^2 - 6 = -6$

$\boxed{(0, -6)}$

$x = 2: -(2)^3 + 3(2)^2 - 6 = -2$

$\boxed{(2, -2)}$

For each problem, find the indicated derivative with respect to x.

18) $f(x) = -5x^4 - 5x^3 + 5x^2$ Find f''

$f' = -20x^3 - 15x^2 + 10x$

$\boxed{f'' = -60x^2 - 30x + 10}$

19) $f(x) = 2x^4 + 3x^2 + 5x$ Find $f''(2)$

$f' = 8x^3 + 6x + 5$

$f'' = 24x^2 + 6$

$f''(2) = 24(2)^2 + 6$

$24(4) + 6 = \boxed{102}$