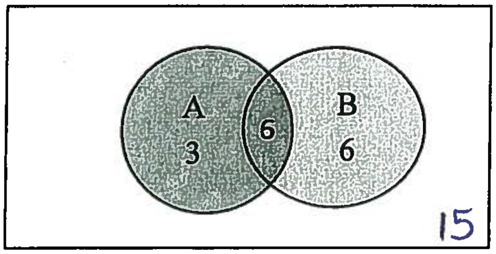
Section 3.4
Conditional Probability

Let's look again at the Venn diagram showing students who have Mr. Kaiser and Mr. Mumaw.



If we know a particular student has Mr. Kaiser, how does this affect the probability that they also have Mr. Mumaw?

Altogether 9 students have Mr. Kaiser; of these 6 also have Mr. Mumaw.

We write the probability that a student has Mr. Mumaw given that they have Mr. Kaiser as $P(B|A)$ (meaning probability of B given that A has occurred)

B and A

So, $P(B|A) = \dfrac{n(B \cap A)}{n(A)} = \dfrac{6}{9} = \dfrac{2}{3}$

This is known as <u>conditional probability</u> since the outcome of *A* is **dependent** on the outcome of *B.*

It also follows that $P(B|A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{6/30}{9/30} = \dfrac{6}{9} = \dfrac{2}{3}$

In general for two events *A* and *B* the probability of *A* occurring given that *B* has occurred can be found using

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

And if we do some math we get $P(A \cap B) = P(A|B) \cdot P(B)$

And this leads to

If *A* and *B* are independent events,

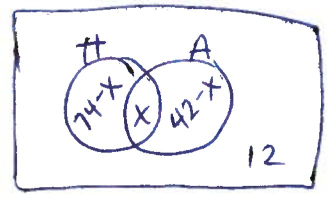
$$P(A|B) = P(A), P(B|A) = P(B), P(A|B') = P(A), \text{ and } P(B|A') = P(B)$$

Since $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = P(A)$

Example: Of the 114 faculty members at NAHS, 74 use an IPhone, 42 use an IPad, and 12 use neither. How many staff use and IPhone and an IPad? $x=14$

One member of the faculty is chosen at random. Find the probability that:

a. He uses an IPhone but not an IPad.
b. If he is an IPhone user he also uses an IPad
c. If he uses an IPad he does not use an IPhone.



Let H = IPhone
A = IPad
x = both

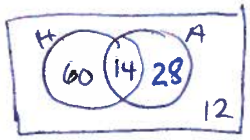
$74-x+x+42-x+12=144$

$128-x=114$

$x=14$

or $74+42+12=128$

$128-114=14$



a) $P(H \cap A') = \frac{60}{114} = \frac{10}{19}$

b) $P(A \mid H) = \frac{14}{74} = \frac{7}{37}$

or $\frac{P(A \cap H)}{P(H)} = \frac{\frac{14}{114}}{\frac{74}{114}} = \frac{14}{74}$

c) $P(H' \mid A) = \frac{28}{42} = \frac{2}{3}$ or $\frac{P(H' \cap A)}{P(A)} = \frac{28/114}{42/114} = \frac{28}{42} = \frac{2}{3}$

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<u>Section 3.5</u>
<u>Probability tree diagrams</u>

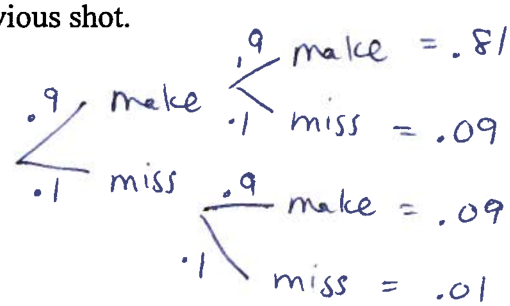
Tree diagrams are useful for problems where more than one event occurs.

- It is easier to use than to list all the possible outcomes.

- With Replacement and repeated events

Example: The probability that Yogi Ferrell makes a free throw is 0.9. He takes 2 shots. Assume that each shot is independent from the previous shot.

Represent this situation in a tree diagram.



notice
Sum = 1

Find the probability that Yogi

a. Makes both free throws.

$P(\text{make} \,\&\, \text{make}) = .9 \times .9 = .81$

b. Makes only one free throw.

$P(\text{make} \,\&\, \text{miss}) + P(\text{miss} \,\&\, \text{make}) = .09 + .09 = .18$

c. Misses both free throws.

$P(\text{miss} \,\&\, \text{miss}) = .1 \times .1 = .01$

d. Makes at least one free throw.

$1 - P(\text{miss} \,\&\, \text{miss}) =$

$1 - .01 = .99$ or part a + part b

$.81 + .18 = .99$