Ch. 7 Non-Calc Review

1. (a) $4x^3+3x^2-2x+6$ → $\boxed{12x^2+6x-2}$

(b) $x^{4/3}$

$\frac{4}{3}x^{1/3}$ or $\frac{4}{3}\sqrt[3]{x}$ or $\frac{4\cdot\sqrt[3]{x}}{3}$ or $\frac{4x^{\frac{1}{3}}}{3}$

(c) $3x^{-4}$

$-12x^{-5} = \boxed{\frac{-12}{x^5}}$

(d) PR $(x^2-1)(2x^3-x^2+x)$

$(x^2-1)(6x^2-2x+1)+(2x^3-x^2+x)(2x)$

$6x^4-2x^3+x^2-6x^2+2x-1+4x^4-2x^3+2x^2$

$\boxed{10x^4-4x^3-3x^2+2x-1}$

(e) QR $\frac{(x-4)}{(x+7)} = \frac{(x+7)(1)-(x-4)(1)}{(x+7)^2} = \frac{x+7-x+4}{(x+7)^2} = \boxed{\frac{11}{(x+7)^2}}$

(f) CR $e^{4x} = e^{4x}\cdot 4 = \boxed{4e^{4x}}$

(g) CR $(x^3+1)^4 = 4(x^3+1)^3\cdot 3x^2 = \boxed{12x^2(x^3+1)^3}$

(h) CR $\ln(2x+3) = \frac{1}{2x+3}\cdot 2 = \boxed{\frac{2}{2x+3}}$

(i) QR or PR $\frac{\ln x}{x^2} = \frac{(x^2)(1/x)-(\ln x)(2x)}{x^4} = \frac{x-2x\ln x}{x^4}$

$\boxed{\frac{1-2\ln x}{x^3}}$

(j) $\frac{1}{6}(4x^2-2x) = \frac{1}{6}(8x-2) = \frac{8}{6}x-\frac{2}{6} =$

$\boxed{4/3x-1/3}$

(k) PR $(3x^2+1)(e^x) = (3x^2+1)(e^x)+(e^x)(6x)$

$\boxed{e^x(3x^2+6x+1)}$ ← pulled out e^x as GCF

(l) QR $\frac{2e^x}{e^x-3} = \frac{(e^x-3)(2e^x)-(2e^x)(e^x)}{(e^x-3)^2} \overset{\text{GCF!}}{=} \frac{2e^x(e^x-3-e^x)}{(e^x-3)^2} = \frac{2e^x(-3)}{(e^x-3)^2}$

$\boxed{\frac{-6e^x}{(e^x-3)^2}}$

(m) CR $3(2x-5)^{1/2} = 3/2\,(2x-5)^{-1/2}\cdot 2 = \boxed{\dfrac{3}{(2x-5)^{1/2}}}$ or $\dfrac{3}{\sqrt{(2x-5)}}$

(n) PR/CR $x^2e^{2x} = x^2\cdot e^{2x}\cdot 2 + e^{2x}\cdot 2x$ GCF!

$\boxed{2xe^{2x}(x+1)}$

(o) CR $\ln\left(\frac{1}{x}\right) = \ln(x^{-1}) = \frac{1}{x^{-1}}\cdot(-1x^{-2}) = x\cdot\frac{-1}{x^2} = \boxed{\frac{-1}{x}}$

(2) (a) $(x+h)^3 = (x+h)(x+h)^2 = (x+h)(x^2+2xh+h^2)$

$= x^3+2x^2h+xh^2+hx^2+2xh^2+h^3$

$= \boxed{x^3+3x^2h+3xh^2+h^3}$

(b) $\lim_{h\to 0}\dfrac{[2(x+h)^3-6(x+h)]-[2x^3-6x]}{h}$

$\dfrac{[2(x^3+3x^2h+3xh^2+h^3)-6(x+h)]-[2x^3-6x]}{h}$

$\dfrac{2x^3+6x^2h+6xh^2+2h^3-6x-6h-2x^3+6x}{h}$

$\dfrac{6x^2h+6xh^2+2h^3-6h}{h} = \lim_{h\to 0} 6x^2+6xh+2h^2-6$

$6x^2+6x(0)+2(0)^2-6 = \boxed{6x^2-6}$

(3) $f(x) = 4xe^{x^2-1}$ → derivative requires PR+CR

$f'(x) = 4x(e^{x^2-1}) \cdot 2x + (e^{x^2-1})4$

$8x^2(e^{x^2-1}) + 4(e^{x^2-1})$ GCF

$4e^{x^2-1}(2x^2+1)$

$m_{tangent} = 4e^{1^2-1}(2(1)^2+1) = 4e^0(2+1) = 4(1)(3) = 12$

$m_{normal} = -1/12$

equation $m = -1/12$ pt $(1,4)$ $\boxed{y-4 = -1/12(x-1)}$

(4) find tangent line for $f(x)$

$f'(x) = 6x^2 - 3$

to be parallel to $y = 5x-2$ it must have $m = 5$

so set equation equal to 5 and find what x-values give you a slope of 5

$5 = 6x^2 - 3$

$8 = 6x^2$

$x^2 = 4/3 \qquad x = \pm\sqrt{4/3} = \pm 2/\sqrt{3} = \pm 2\sqrt{3}/3$

now plug those x-values into $f(x)$ to find coordinates

$f(x) = 2x^3 - 3x + 1$

$= 2\left(\frac{2\sqrt{3}}{3}\right)^3 - 3\left(\frac{2\sqrt{3}}{3}\right) + 1$

$= 2\left(\frac{2\sqrt{3}}{3}\right)\left(\frac{2\sqrt{3}}{3}\right)\left(\frac{2\sqrt{3}}{3}\right) - 2\sqrt{3} + 1$

$= \frac{16\sqrt{3}\sqrt{3}\sqrt{3}}{27} - \frac{54\sqrt{3}}{27} + \frac{27}{27}$

$= \frac{48\sqrt{3} - 54\sqrt{3} + 27}{27}$

$= \frac{27 - 6\sqrt{3}}{27} = \frac{9 - 2\sqrt{3}}{9}$

$= 2\left(\frac{-2\sqrt{3}}{3}\right)^3 - 3\left(\frac{-2\sqrt{3}}{3}\right) + 1$

$= 2\left(\frac{-2\sqrt{3}}{3}\right)\left(\frac{-2\sqrt{3}}{3}\right)\left(\frac{-2\sqrt{3}}{3}\right) + 2\sqrt{3} + 1$

$= -\frac{16\sqrt{3}\sqrt{3}\sqrt{3}}{27} + \frac{54\sqrt{3}}{27} + \frac{27}{27}$

$= \frac{-48\sqrt{3} + 54\sqrt{3} + 27}{27}$

$\frac{27 + 6\sqrt{3}}{27} = \frac{9 + 2\sqrt{3}}{9}$

$\boxed{\left(\frac{2\sqrt{3}}{3}, \frac{9-2\sqrt{3}}{9}\right)\left(\frac{-2\sqrt{3}}{3}, \frac{9+2\sqrt{3}}{9}\right)}$

(7) @ Find average velocity in m/s from $t=1$ to $t=3$

slope of $s(t)$!

$$\frac{[20(3)-100\ln(3)]-[20(1)-100\ln(1)]}{3-1} = \boxed{-34.9 \text{ m/s}}$$

(b) Find instantaneous velocity in m/s at $t=3$

find $v(t)$ at $t=3$

$s(t) = 20t - 100\ln t$

$v(t) = 20 - 100\left(\frac{1}{t}\right)$

$v(3) = 20 - 100\left(\frac{1}{3}\right) = \boxed{-13.3 \text{ m/s}}$

(c) Find speed at $t=3$

$|v(t)| = |v(3)| = |-13.3| = \boxed{13.3 \text{ m/s}}$