

# Solutions

## IB Final Exam Ch 3 Review

$A \cap B$  means the INTERSECTION of A and B (Think A AND B)

$A \cup B$  means the UNION of A and B (Think A OR B)

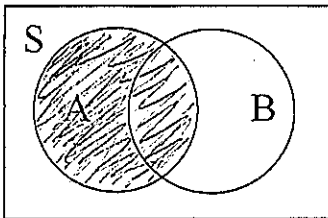
Rectangle 'S' represents a sample space of possible outcomes.

Circles 'A' and 'B' each represent specific unique events in the sample space S.

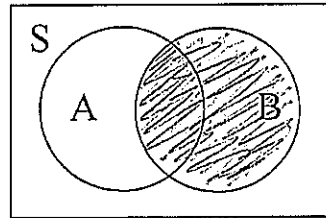
If it is possible for two events A and B to both happen at the same time, then their circles will intersect on a Venn Diagram. **Anything not pertaining to A or B will be shown in rectangle 'S'**

1. Shade the region of the picture corresponding to each listed event.

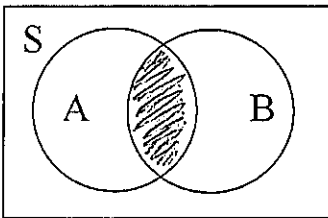
1.) A



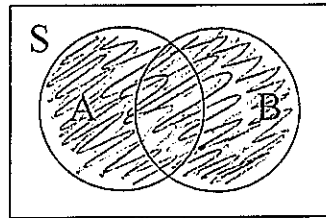
2.) B



3.)  $A \cap B$



4.)  $A \cup B$

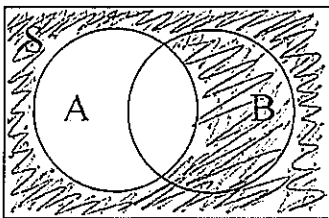


The complement of an event 'A' refers to all outcomes that are NOT included in 'A'.

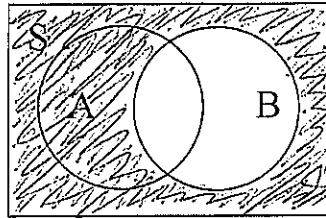
The notation for complement is  $A'$ . The probability of  $A' = 1 - P(A)$ .

Shade the region corresponding to each listed event.

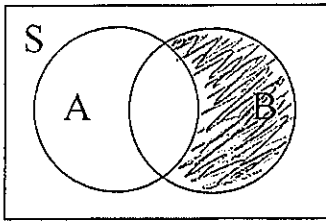
5.)  $A'$



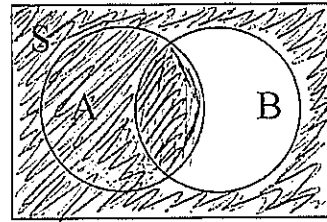
6.)  $B'$



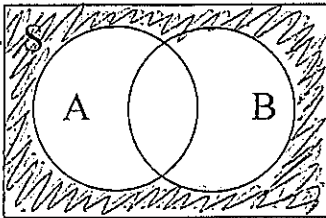
7.)  $A' \cap B$



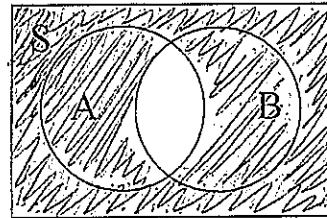
8.)  $A \cup B'$



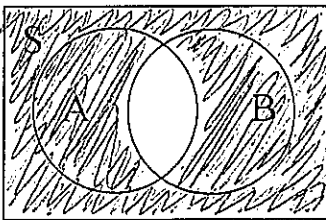
9.)  $A' \cap B'$



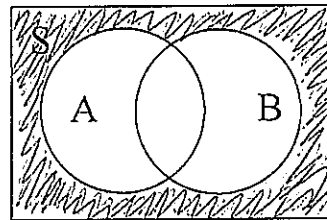
10.)  $A' \cup B'$



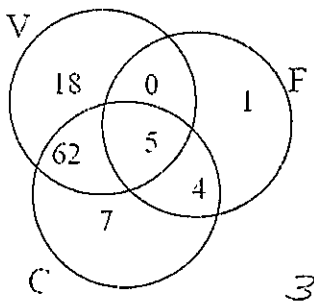
11.)  $(A \cap B)'$



12.)  $(A \cup B)'$



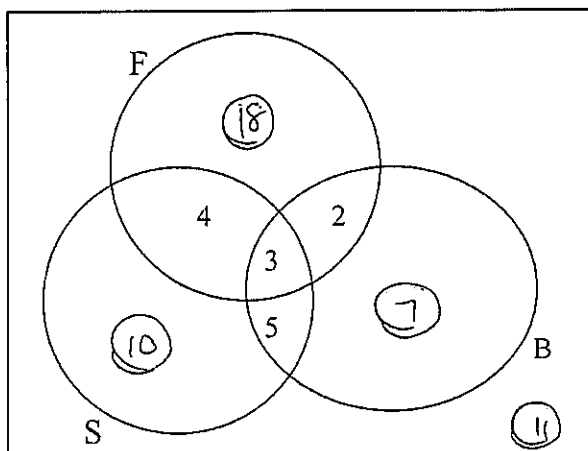
13. The Venn diagram displays the results of a survey of 100 families regarding technology in their homes. Computer (C), VCR (V) and fax machine (F).



How many families have:

- a) a computer at home? 78% (62+5+4+7)
- b) all three machines? 5% (intersection of all 3)
- c) none of the machines in their home? 3% (100 - all #s)
- d) no fax machine? 90% (18+62+7+3)
- e) a computer and a VCR? 67% (62+5)
- f) a VCR or a computer? 96% (18+62+5+7)

14. A group of 60 students were asked if they played field hockey (F), basketball (B) or soccer (S). The diagram below displays the results. 27 students play field hockey, 17 students play basketball, and 22 students play soccer.



What percent of the group play:

- a) field hockey & basketball?  $8.33\%$   $\left(\frac{3+2}{60}\right)$
- b) field hockey or basketball?  $65\%$   $\left(\frac{18+4+2+3+7}{60}\right)$
- c) field hockey & soccer?  $11.67\%$   $\left(\frac{4+3}{60}\right)$
- d) neither of the three sports?  $18.33\%$   $\left(\frac{60 - \text{all #s include}}{60}\right)$
- e) only 1 sport?  $58.33\%$   $\left(\frac{18+10+7}{60}\right)$

15. A die is rolled twice. Find each probability as a reduced fraction.

a.  $P(\text{twos 4s})$

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

b.  $P(2 \text{ and } 3)$

$$2 \left(\frac{1}{6} \cdot \frac{1}{6}\right) = \frac{1}{18}$$

c.  $P(\text{no 6s})$

$$\frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$$

d.  $P(\text{at least one 3})$

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \quad (2 \text{ 3's})$$

$$+ 2 \left(\frac{1}{6} \cdot \frac{5}{6}\right) = \frac{10}{36} \quad \left(\frac{11}{36}\right)$$

16. There are 8 action, 3 comedy, and 5 drama DVDs on a shelf. Suppose three DVDs are selected at random from the shelf. Find each probability as a reduced fraction.

a. three action movies, with replacement  $\frac{8}{16} \cdot \frac{8}{16} \cdot \frac{8}{16} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

b. 2 action then a comedy without replacement

$$\frac{8}{16} \cdot \frac{7}{15} \cdot \frac{3}{14} = \frac{168}{3360} = \frac{1}{20}$$

17. There are 20 pieces of candy in a bag, 15 are chocolate and 5 are hard candy. Two pieces are selected at random and eaten. Find each probability as a reduced fraction.

a. Both are hard candy  $\frac{5}{20} \cdot \frac{4}{19} = \frac{20}{380} = \frac{1}{19}$

b. One chocolate and one hard candy

$$\frac{15}{20} \cdot \frac{5}{19} + \frac{5}{20} \cdot \frac{15}{19} = \frac{15}{76} + \frac{15}{76} = \frac{30}{76} = \frac{15}{38}$$

c. At least one hard candy

$$\rightarrow \text{prob of a + b} = \frac{1}{19} + \frac{15}{38} = \frac{17}{38}$$

**Answers:**

- 13. a) 78%    b) 5%    c) 3%    d) 90%    e) 67%    f) 96%
- 14. a) 8.33%    b) 65%    c) 11.67%    d) 18.33%    e) 58.33%
- 15. a) 1/36    b) 1/18    c) 25/36    d) 11/36
- 16. a) 1/8    b) 1/20
- 17. a) 1/19    b) 15/38    c) 17/38

## SIMPLE STATS SOLUTIONS

1.

$$(a) \quad \frac{a+6}{2} = 5$$

$$a+6=10$$
$$a=4$$

$$(b) \quad \frac{42+a+b}{8} = 6$$

$$42+a+b=48$$

$$a+b=6$$

$$4+b=6$$

$$b=2$$

2. Total exams:  $80+60=140$     Sum of Scores:  $(80 \times 62.1) + (60 \times 56.8) = 8376$

New Mean:  $8376 \div 140 = 59.8$  (3 sf)

3.  $(220 + 300 + 210 + 275 + 240 + x) \div 6 = 250$

$$x + 1245 = 1500 \rightarrow x = 255 \text{ km}$$

4.

(i) Median = 10

(ii)  $Q_3 = 12$

(iii) Min value =  $1(\pm 0.2)$

5. Mean: 5.11 (3 sf); Median: 5; Mode: 3; Range: 9 (Max-Min); SD: 3.14; IQR: 5.5 ( $Q_3-Q_1$ )

6. (a) Mean: 2.57 (3 sf); Median: 2; Mode: 1; SD: 1.72 (3 sf)

(b) Range: 6 (Max-Min); Lower Quartile ( $Q_1$ ): 1; IQR: 3 ( $Q_3-Q_1$ )

7. (a) 19 or 20 people

(b) Median salary = 15000 GBP

(c) 80% of 200 = 23000 ish

8. (a)  $y = 20.9 - 0.134x$

(b) 17 objects (must be a whole number)

(c)  $r = -0.756$

(d) moderately strong, negative correlation

## COMPLEX STATS SOLUTIONS

1. (a)
  - (b) (i) Between  $\pm 1$  standard deviations... 68% or 0.68
  - (ii) Less than 103 = 50% plus 34% = 84% or 0.84
  - (iii) Between 87 & 111 = 68% + 13.5% = 81.5% or 0.815
  - (iv) More than 95 cm = 50%
  
2. (a) using graphing calc:  $P(X < 130) = \text{Normalcdf}(-10000, 130, 100, 20) = 0.933$   
(b) using graphing calc:  $P(X > 90) = \text{Normalcdf}(90, 10000, 100, 20) = 0.691$   
(c) using graphing calc:  $P(80 < X < 125) = \text{Normalcdf}(80, 125, 100, 20) = 0.736$
  
3.  $P(X < x) = 0.90$  using graphing calc:  $\text{invNorm}(0.9, 100, 20) = \$126$  (3 sf)
  
4. (a) (i)  $H_0$ : There is no connection between gender and subject taken.  
(ii) Degrees of freedom =  $(3 - 1)(2 - 1) = 2 \times 1 = 2$   
(iii)  $p = 0.41$   
(iv) Since  $0.41 > 0.05$  (5% significance), Accept  $H_0$   
Remember... if the probability ( $p$ ) is low, reject
  
5. (a) Expected number of male managers  
$$= \frac{160}{500} \times \frac{300}{500} \times 500 = \frac{160 \times 300}{500}$$
$$= 96$$
  
(b) (i)  $H_0$ : Position is independent of gender  
(ii)  $H_1$ : Position is dependent on gender  
(c) (i)  $\chi^2 = 12.8$   
(ii) Since  $12.8 > 5.99$  (critical value), Reject  $H_0$   
Remember... if Chi is high, reject

### LOGIC Solutions

1. (a) *"If the water is not cold and not boiling then it is warm"*  
(or equivalent statement)
- (b)  $p \Leftrightarrow \neg(q \vee r)$  or  $p \Leftrightarrow \neg q \wedge \neg r$

2. (a) (i) "The food supply is adequate and the visitors are hungry but the oven is not working," (or equivalent statement).
- (ii) "Either the oven is working and the food supply is adequate or the visitors are not hungry," (or equivalent statement).
- (b)  $(p \wedge q) \Rightarrow (p \vee q)$

3.

$p$	$q$	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$(p \wedge q) \Rightarrow (p \vee q)$  is a tautology

4. (a)

$p$	$q$	$p \Leftrightarrow q$	$(p \Leftrightarrow q) \wedge p$	$[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	T

(b) The statements are logically equivalent.

5. (a) (i) *If you do not watch the music TV channel, then you do not like music.*
- (ii) *If you like music, then you watch the music TV channel.*
- (iii) *If you don't like music, then you don't watch the music TV channel.*

# Numbers & Algebra Final Exam Rev. Solution

1) (a)  $A = \text{set of integers}$   
 $\{-4, 1, 13, 67, 10^{33}\}$

(b)  $B = \text{set of rationals}$   
 $\{-4, -2/3, 1, 13, 26.7, 67, 10^{33}\}$

(c)  $A \cap B = \text{\#s in both sets}$   
 $\{-4, 1, 13, 67, 10^{33}\}$

2) (a)  $1.623 \text{ m} \rightarrow \text{cm (nearest)}$   
 $(1.623)(100) = 162.3 \Rightarrow \boxed{162 \text{ cm}}$

(b)  $2 \text{ hrs } 43 \text{ min} \rightarrow \text{nearest 5 min}$   
 $\boxed{2 \text{ hrs } 45 \text{ min}} \text{ or } \boxed{165 \text{ min}}$

(c)  $2591 \text{ people} \rightarrow \text{nearest 100 people}$   
 $\boxed{2600 \text{ people}}$

(d)  $384\,403\,000 \text{ m} \rightarrow \text{km}$   
 $\div 1000$   
 $384\,403 \text{ km} = \boxed{3.84403 \times 10^5}$   
or  $\boxed{3.84 \times 10^5}$

(e)  $1 \text{ week} = 7 \text{ days}$

$$7(1250) = 8750 \text{ mm}$$

$$\text{mm} \rightarrow \text{m} = 1000 \text{ mm} = 1 \text{ m}$$

$$8750 \text{ mm} \div 1000 = \boxed{8.75 \text{ m}}$$

$$3) \frac{300 \text{ m}}{1 \text{ sec}} = \frac{x}{60 \text{ sec (1 hr)}} \quad \text{a) } x = 18000 \text{ m h}^{-1}$$

$$\text{b) (i) } 18000 \text{ m h}^{-1}$$

$$\text{(ii) } 1.8 \times 10^4 \text{ m h}^{-1}$$

$$4) \text{ a) } p = (24.15)^2 + 2(3.6)(24.15) = 757.1025$$

$$\text{b) } 757.10$$

$$\text{c) } 2 \text{ sf} \Rightarrow 760$$

$$\text{d) } \varepsilon = \left| \frac{V_A - V_E}{V_E} \right| \times 100\% \quad V_A = 760 \quad V_E = 757.1025$$

$$\left| \frac{760 - 757.1025}{757.1025} \right| \times 100\% = 0.383\% \quad (3 \text{ sf}) \quad \text{Always positive!}$$

$$5) \text{ a) Area} = (91.4)(68.5) = 6260.9 \text{ or } 6260 \text{ m}^2 \quad (3 \text{ sf})$$

b) Area in  $\text{cm}^2$

$$6260.9 \times 100 \times 100 = 62609000 \text{ or } 62600000 \text{ cm}^2$$

$$6260 \times 100 \times 100 = 62600000 \text{ cm}^2$$

$$6) \$1 = .5417 \text{ UK pounds}$$

$$560(.5417) = 303.35 \text{ UK pounds}$$

$$7) \$1 \text{ CAD} = \pounds 0.4073$$

$$1 \pounds = \frac{1}{0.4073} \text{ CAD}$$

$$700 \left( \frac{1}{0.4073} \right) = 1718.63 \text{ CAD}$$

$$8) 1 \text{ Swiss franc} = 0.6807 \text{ EURO} \quad 1.8\% \text{ Commission}$$

$$(1200)(0.6807) = 816.84 \cdot .982$$

$$802.14 \text{ EURO}$$



9) a) Buy 1 peso = 3.4807 baht  
 $(400)(3.4807) = \boxed{1392.28 \text{ baht}}$

b) Sell 1 peso = 3.584  
 $\frac{1392.28}{3.584} = \boxed{388.47 \text{ peso}}$

c)  $400 - 388.47 = \boxed{11.53 \text{ peso}}$

10) Let C = CDs V = video cassettes

a)  $6C + 3V = 163.17$   
 $9C + 2V = 200.53$

b) Solve  $\rightarrow$  use app PlySmlt2

$$C = 367/20 = \$18.35$$

$$V = 1769/100 = \boxed{\$17.69} \leftarrow \text{price of video}$$

c)  $9 \text{ CDs} = 9(18.35) = \$165.15$   
 $180 - 165.15 = \boxed{\$14.85}$

11) a)  $x^2 - 5x - 24 = 0$   
 $(x - 8)(x + 3) = 0$   
 $\boxed{x = 8 \quad x = -3}$

b)  $x^2 - 5x - 20 = 0$   
use app PlySmlt2  
 $\boxed{x = -2.62 \text{ or } 7.62}$   
(3 sf)

12)  $(x - 4)(2x) = 10$

$$2x^2 - 8x - 10 = 0$$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

$$\boxed{x = 5} \quad x = -1 \leftarrow \text{can't have negative side}$$

## GEOMETRY & TRIG SOLUTIONS

1. (a) gradient =  $\frac{-4}{3}$  or -1.33(3 s.f.)

(b)  $y = \frac{-4}{3}x + 4$

OR  $4x + 3y - 12 = 0$

OR equivalent form

(c)  $y = \frac{-4}{3}x - 4$

$4x + 3y + 12 = 0$  or  $\frac{4}{3}x + y + 4 = 0$

2. (a)  $9x + 12 = 3y \rightarrow 3x + 4 = y \rightarrow \text{Slope} = 3$

(b)  $-1/3$

(c)  $y = \frac{-1}{3}x + 9$  or equivalent

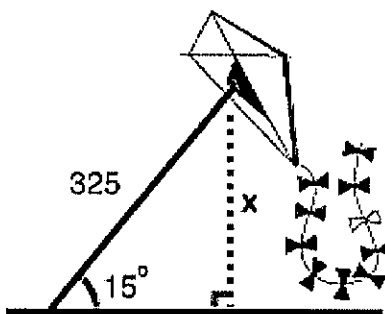
(d) (1.5, 8.5)

3. (a) (i) (2, -2)

(ii)  $\sqrt{(0-4)^2 + (-5-1)^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$  or 7.21

(iii) gradient of PQ =  $\left(\frac{-5-1}{0-4}\right) = \frac{6}{4} = \frac{3}{2}$  (1.5)

4.



$$\sin 15^\circ = \frac{x}{325}$$

84.1 feet

5. (a)  $AC^2 = 625^2 + 986^2 - 2 \times 625 \times 986 \times \cos 102^\circ$   
 $AC^2 = 1619072.159$   
 $AC = 1272.43$   
 $= 1270\text{m}$

(b)

$$\frac{986}{\sin A} = \frac{1270}{\sin 102^\circ} \quad \text{OR} \quad \frac{986}{\sin A} = \frac{1272.43}{\sin 102^\circ} \quad \text{OR} \quad \cos A = \left( \frac{625^2 + 1270^2 - 986^2}{2 \times 625 \times 1270} \right)$$

$$A = 49.4^\circ$$

$$A = 49.3^\circ$$

$$A = 49.5^\circ$$

(c)  $A = \frac{1}{2}(625)(986) \sin 102^\circ = 301,391.73 \text{ m}^2$

6. (a) (i) Angle  $\hat{BTL} = 180 - 80 - 26.5$  or  $180 - 90 - 26.5 + 10 = 73.5^\circ$

(ii)  $\frac{BT}{\sin(26.5^\circ)} = \frac{120}{\sin(73.5^\circ)}$   
 $BT = 55.8 \text{ m (3s.f.)}$

(b)  $TG = 55.8 \sin(80^\circ)$  or  $55.8 \cos(10^\circ) = 55.0 \text{ m (3s.f.)}$

7. Volume of the Prism = Area of the Triangular Base times the Height (distance between the bases)

Area of Base: Height of Triangle (h):  $\tan 42^\circ = \frac{h}{5.7} \rightarrow h = 5.13$

$$\text{Area of Triangle} = \frac{1}{2}bh = \frac{1}{2}(5.7)(5.13) = 14.6$$

Volume: Area of Base times Height =  $(14.6)(8) = 116.8$  or  $117 \text{ cm}^3$

Surface Area of Prism = 2 Triangles plus bottom rectangle plus side rectangle plus slanted rectangle

First I need the hypotenuse of the triangle:  $\cos 42^\circ = \frac{5.7}{x} \rightarrow x = 7.67$

Surface Area =  $2\left(\frac{1}{2}(5.7)(5.13)\right) + (5.7)(8) + (5.13)(8) + (7.67)(8) = 177.2 \text{ cm}^2$  (exact) or  $177 \text{ cm}^2$  (3 s.f.)

8. To find the surface area of the cone, find the slant height. The radius of the cone, cylinder, and sphere is 3 cm.

$$\text{Slant Height } (l) \rightarrow l^2 = 3^2 + 5^2 \rightarrow l = \sqrt{34}$$

The height of the semi-sphere is the radius... 3 cm. So the height of the cylinder is  $16 - 5 - 3 = 8$  cm.

Surface Area (SA) = SA of Cone (minus bottom) + SA of Cylinder (minus top & bottom) + SA of Half a Sphere.

$$= \pi r l + 2\pi r h + \frac{1}{2}(4\pi r^2) = \pi(3)(\sqrt{34}) + 2\pi(3)(8) + 2\pi(3^2) = 262.3 \text{ or } 262 \text{ cm}^2$$

Volume: Cone + Cylinder +  $\frac{1}{2}$  (Sphere)

$$\text{Volume: } \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{3}\pi(3)^2(5) + \pi(3)^2(8) + \frac{1}{2}\left(\frac{4}{3}\pi(3)^3\right) = 330 \text{ cm}^3$$

9. (a)  $V = \frac{1}{3} \times 3.2^2 \times 2.8 = 9.56 \text{ cm}^3$

(b)  $9.56 \times 9.3 = 88.9 \text{ grams}$

(c)

$$\frac{1}{2} \text{ base} = 1.6 \text{ seen}$$

$$OC^2 = 1.6^2 + 1.6^2 = 5.12$$

$$5.12 + 2.8^2 = 12.96 = VC^2$$

$$VC = 3.6$$

OR

$$AC^2 = 3.2^2 + 3.2^2 = 20.48$$

$$OC = \frac{1}{2}\sqrt{20.48} (=2.26\dots)$$

$$2.8^2 + (2.26\dots)^2 = VC^2 = 12.96$$

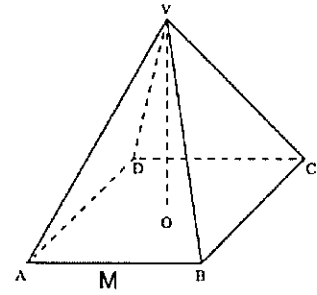
$$VC = 3.6$$

(d)  $4 \times \frac{1}{2}(3.6)^2 \times \sin(52.8^\circ) + (3.2)^2$   
 $= 30.9 \text{ cm}^2$

See next page for part e

- (e) Think of the triangle made by points VMO where M is the midpoint of side AB. Segment OM is 1.6 cm (half the length of the side). And I am given the height of the triangle is 2.8. Use trig to find angle VMO

$$\tan x = \frac{2.8}{1.6} \rightarrow \tan^{-1}\left(\frac{2.8}{1.6}\right) = 60.3^\circ$$



10. (a) AC is the hypotenuse of Right Triangle ABC  
 $AC^2 = (7.2)^2 + (9.6)^2 \rightarrow AC = 12 \text{ m}$
- (b) AG is the hypotenuse of Right Triangle ACG  
 $AG^2 = (AC)^2 + (CG)^2 \rightarrow AG^2 = (12)^2 + (3.5)^2 \rightarrow AG = 12.5 \text{ m}$
- (c) Calculate the angle that AG makes with the floor.

Since you know all three sides of a right triangle, you can do an inverse trig function of any function you wish. Let's use cosine just because...

$$\cos \theta = \frac{12}{12.5}$$

$$\cos^{-1}\left(\frac{12}{12.5}\right) = \theta$$

$$\theta = 16.3^\circ$$

