

Probability Review

$A \cap B$ means the INTERSECTION of A and B (Think A AND B)

$A \cup B$ means the UNION of A and B (Think A OR B)

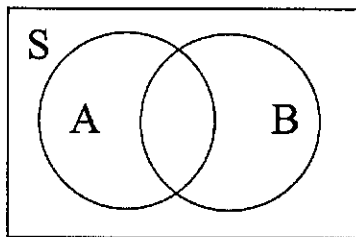
Rectangle 'S' represents a sample space of possible outcomes.

Circles 'A' and 'B' each represent specific unique events in the sample space S.

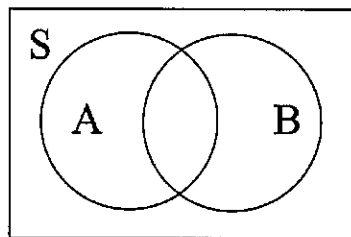
If it is possible for two events A and B to both happen at the same time, then their circles will intersect on a Venn Diagram. **Anything not pertaining to A or B will be shown in rectangle 'S'**

I. Shade the region of the picture corresponding to each listed event.

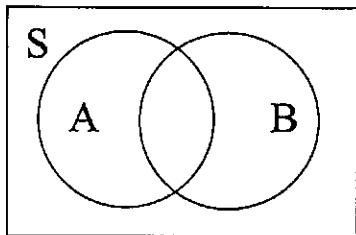
1.) A



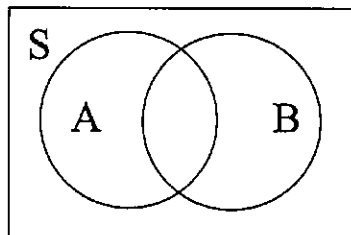
2.) B



3.) $A \cap B$



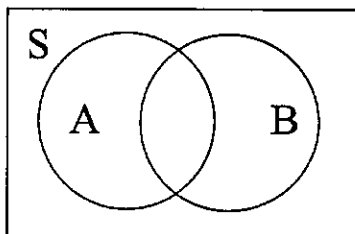
4.) $A \cup B$



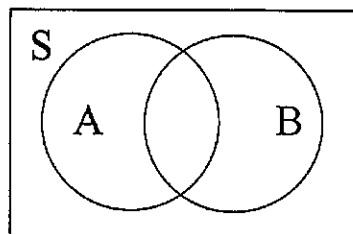
The complement of an event 'A' refers to all outcomes that are NOT included in 'A'.
The notation for complement is A' . The probability of $A' = 1 - P(A)$.

Shade the region corresponding to each listed event.

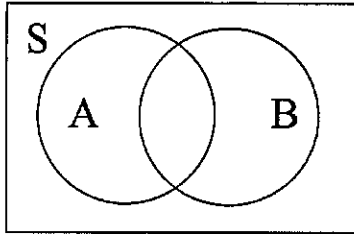
5.) A'



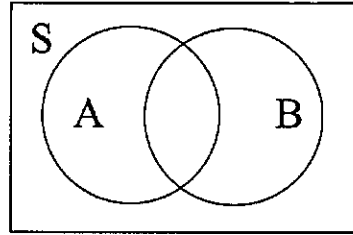
6.) B'



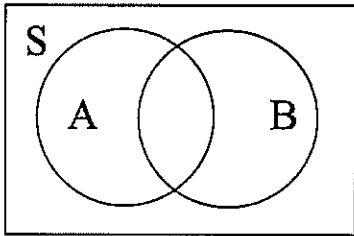
7.) $A' \cap B$



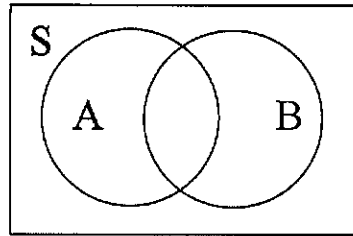
8.) $A \cup B'$



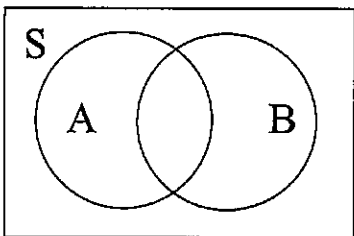
9.) $A' \cap B'$



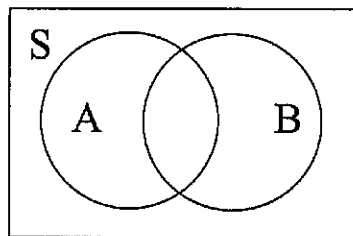
10.) $A' \cup B'$



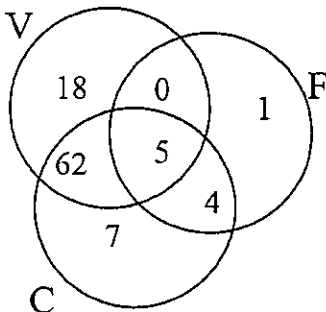
11.) $(A \cap B)'$



12.) $(A \cup B)'$



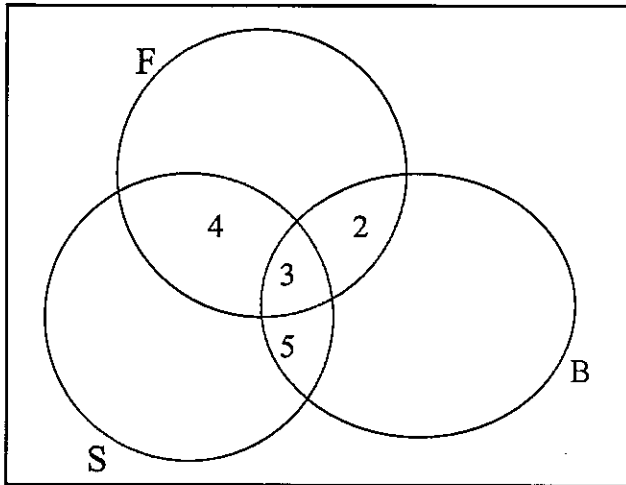
13. The Venn diagram displays the results of a survey of 100 families regarding technology in their homes. Computer (C), VCR (V) and fax machine (F).



How many families have:

- a computer at home?
- all three machines?
- none of the machines in their home?
- no fax machine?
- a computer and a VCR?
- a VCR or a computer?

14. A group of 60 students were asked if they played field hockey (F), basketball (B) or soccer (S). The diagram below displays the results. 27 students play field hockey, 17 students play basketball, and 22 students play soccer.



- What percent of the group play:
- field hockey & basketball?
 - field hockey or basketball?
 - field hockey & soccer?
 - neither of the three sports?
 - only 1 sport?

15. A die is rolled twice. Find each probability as a reduced fraction.

- P (two 4s)
- P (2 and 3)
- P (no 6s)
- P (at least one 3)

16. There are 8 action, 3 comedy, and 5 drama DVDs on a shelf. Suppose three DVDs are selected at random from the shelf. Find each probability as a reduced fraction.

- three action movies, with replacement
- 2 action then a comedy without replacement

17. There are 20 pieces of candy in a bag, 15 are chocolate and 5 are hard candy. Two pieces are selected at random and eaten. Find each probability as a reduced fraction.

- Both are hard candy
- One chocolate and one hard candy
- At least one hard candy

Answers:

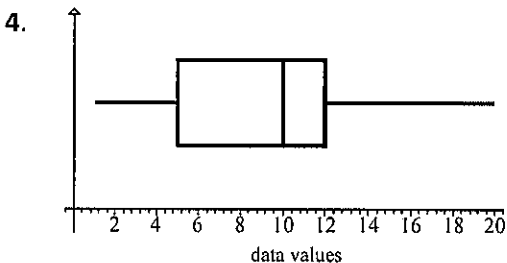
13. a) 78% b) 5% c) 3% d) 90% e) 67% f) 96%
14. a) 8.33% b) 65% c) 11.67% d) 18.33% e) 58.33%
15. a) 1/36 b) 1/18 c) 25/36 d) 11/36
16. a) 1/8 b) 1/20
17. a) 1/19 b) 15/38 c) 17/38

SIMPLE STATS

1. In the following ordered data, the mean is 6 and the median is 5. Find the value of a and b .

$2, b, 3, a, 6, 9, 10, 12$

2. Peter has graded 80 exams. He calculated the mean score for the exams to be 62.1 marks. Maria has graded 60 exams with a mean score of 56.8 marks. Peter and Maria put all their exams together. Calculate the value of the mean for all the exams.
3. A family must drive an average of 250 km a day to finish vacation on time. On the first five days they drive 220 km, 300 km, 210 km, 275 km, and 240 km. How many km must they drive on the last day to finish on time?



The boxplot at the right shows the statistics for a set of data.

For this data set write down the value of

- (i) the median;
- (ii) the upper quartile;
- (iii) the minimum value present.

5. Find the mean, median, mode, range, standard deviation and IQR of the following numbers
1, 7, 9, 2, 3, 6, 5, 10, 3
6. The numbers of hours students study mathematics each night is given in the table below.

Hours	0	1	2	3	4	5	6
f	2	5	4	3	4	2	1

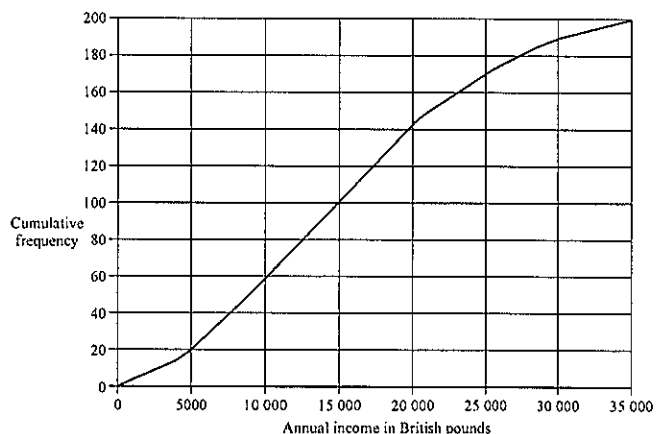
- a) Find the mean, median, mode, and standard deviation.
- b) Find the range, lower quartile, and interquartile range.

7.

The graph at the right shows the cumulative frequency for the yearly incomes of 200 people.

Use the graph to estimate

- a) the number of people who earn less than 5000 British pounds per year;
- b) the median salary of the group of 200 people;
- c) the income of the 80th percentile.



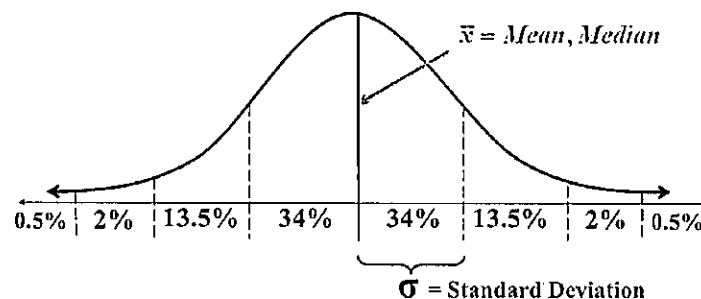
8. Tania wishes to see whether there is any correlation between a person's age and the number of objects on a tray which could be remembered after looking at them for a certain time. She obtains the following table of results.

Age (x years)	15	21	36	40	44	55
Number of objects remembered (y)	17	20	15	16	17	12

- Use your graphic display calculator to find the equation of the regression line of y on x .
- Use your equation to estimate the number of objects remembered by a person aged 28 years.
- Use your graphic display calculator to find the correlation coefficient r .
- Comment on your value for r .

COMPLEX STATS

1. The chest measurements of 18 year old male soccer players are normally distributed with a mean of 95 cm and a standard deviation of 8 cm. Use the 68-95-99% Rule diagram below as a guide to answer the following questions.



- Draw a normal distribution curve for the soccer player scenario. Label it with the mean and 1, 2, and 3 standard deviations above and below the mean.
 - Using the diagram above as a guide, find the percentage of players with chest measurements
 - between 87 & 103 cm
 - more than 95 cm
 - less than 103 cm
 - between 87 & 111 cm
2. Households in Portugal spend an average of 100 Euros per week on groceries with a standard deviation of 20 Euros. Assuming that the distribution of grocery expenditure follows a normal distribution, what is the probability of a household spending:
- less than 130 Euros per week
 - more than 90 Euros per week
 - Between 80 Euros and 125 Euros per week
3. In the scenario above, what spending amount would fall in the 90th percentile (probability of spending at that amount or lower is 90%)

4. For his Mathematical Studies Project a student gave his classmates a questionnaire to fill out. The results for the question on the gender of the student and specific subjects taken by the student are given in the table below, which is a 2×3 contingency table of **observed** values.

	History	Biology	French
Female	22	20	18
Male	20	11	9

The chi-squared test is used to determine if the choice of subject is independent of gender, at the 5% level of significance.

- (a) (i) State a suitable null hypothesis H_0 .
- (ii) Show that the number of degrees of freedom is two.
- (iii) Use the χ^2 Test to find the p value.
- (iii) Based on the p value and the 5% level of significance, do you accept H_0 ? Explain your answer.

5. A survey was conducted in a company to determine whether position in upper management was independent of gender. The results of this survey are tabulated below.

	Managers	Junior executives	Senior executives	Totals
Male	95	130	75	300
Female	65	110	25	200
Totals	160	240	100	500

- (a) Show that the expected number of **Male Managers** is 96.
- (b) (i) Write a suitable null hypothesis for this data.
- (ii) Write a suitable alternate hypothesis for this data.
- (c) (i) Perform a chi-squared test of independence for this data. Find the calculated value of χ^2 .
- (ii) The critical value of χ^2 at the 5% significance level is 5.99. Based on this value, do you accept H_0 ? Explain your answer.

LOGIC

1. Three propositions p , q and r are defined as follows:
 p : the water is cold. q : the water is boiling. r : the water is warm.

(a) Write one sentence, in words, for the following logic statement:
 $(\neg p \wedge \neg q) \Rightarrow r$

(b) Write the following sentence as a logic statement using symbols only.
"The water is cold if and only if it is neither boiling nor warm"

2. Three propositions are defined as follows:
 p : The oven is working. q : The food supply is adequate. r : The visitors are hungry.

(a) Write one sentence, in words only, for each of the following logic statements.

(i) $q \wedge r \wedge \neg p$ (ii) $\neg r \vee (p \wedge q)$

(b) Write the sentence below using only the symbols p , q and logic connectives.

"If the oven is working and the food supply is adequate then the oven is working or the food supply is adequate."

3. Complete the truth table below. Then describe the last column as a tautology, a contradiction, or neither.

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \Rightarrow (p \vee q)$
T	T			
T	F			
F	T			
F	F			

4. (a) Complete the truth table below.

(b) Compare the last column with the last column above. Are the two statements logically equivalent?

p	q	$p \Leftrightarrow q$	$(p \Leftrightarrow q) \wedge p$	$[(p \Leftrightarrow q) \wedge p] \Rightarrow q$
T	T	T		T
T	F		F	T
F	T	F		T
F	F	T	F	

5. Let p and q be the statements
 p : you watch the music TV channel q : you like music

(a) Consider the following logic statement.

If you watch the music TV channel then you like music.

Write down in words the inverse, converse, and contrapositive of the statement.

Numbers and Algebra

1. Let $U = \{-4, -\frac{2}{3}, 1, \pi, 13, 26.7, 67, 10^{33}\}$.

A is the set of all the integers in U .

B is the set of all the rational numbers in U .

- (a) List all the members of A .
- (b) List all the members of B .
- (c) List all the members of the set $A \cap B$.

2. (a) A girl's height is 1.623 m. Write her height **to the nearest cm**.
- (b) The time taken to fill a tank was 2 hours 43 minutes. Write this time **to the nearest 5 minutes**.
- (c) The attendance at a show was 2591 people. How many people, **to the nearest 100**, were at the show?
- (d) The mean distance of the Moon from the Earth is approximately 384 403 000 m. Write this distance in **kilometers** in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.
- (e) An inchworm travels 1250 millimeters a day. How far to the nearest **m** does it travel in a week?

3. The speed of sound in air is given as 300 ms^{-1} .

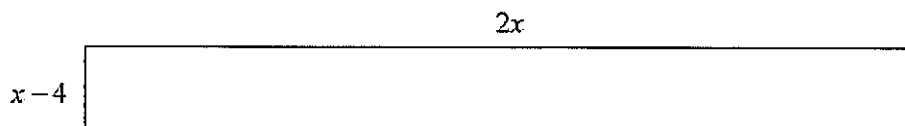
- (a) How many metres does sound travel in air in one hour?
- (b) Express your answer to part (a)
 - (i) correct to **two** significant figures;
 - (ii) in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

4. Given the equation $p = r^2 + 2qr$,

- a) Calculate the exact value of p if $q = 3.6$ and $r = 24.15$.
- b) Write your answer correct to two decimal places.
- c) Write your answer correct to two significant figures.
- d) Find the percentage error between a) and c).

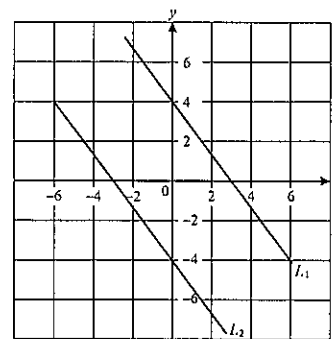
5. A field is 91.4 m long and 68.5 m wide.
- Calculate the area of the field in m^2 .
 - Calculate the area of the field in cm^2 .
6. If \$1 buys 0.5417 UK pounds, find how many UK pounds could be bought for \$560?
7. What does it cost in Canadian dollars to buy £700, given that \$1 Canadian = £0.4073 ?
8. A bank exchanges 1200 Swiss francs to euro for a commission of 1.8%. What does the customer receive for the transaction if 1 Swiss franc = 0.6807 euro?
9. A currency exchange service in Thailand exchanges pesos for Thai baht using:
“buy at 3.4807, sell at 3.584.”
- Sergio wishes to exchange 400 pesos for Thai baht. How many baht will he receive?
 - If he immediately converted the baht back to pesos, how many pesos would he get?
 - What is the resulting commission on the double transaction?
10. Jacques can buy six CDs and three video cassettes for \$163.17 or he can buy nine CDs and two video cassettes for \$200.53.
- Express the above information using two equations relating the price of CDs and the price of video cassettes.
 - Find the price of one video cassette.
 - If Jacques has \$180 to spend, find the exact amount of change he will receive if he buys 9 CDs.
11.
 - Find the solution of the equation $x^2 - 5x - 24 = 0$
 - Find the solution of the equation $x^2 - 5x = 20$
12. The surface of a red carpet is shown below. The dimensions of the carpet are in metres. And the area of the carpet is 10 m^2 . Find the value of x .

diagram not to scale



GEOMETRY & TRIG

1. In the diagram, the lines L_1 and L_2 are parallel.
- What is the gradient of L_1 ?
 - Write down the equation of L_1 .
 - Write down the equation of L_2 in the form $ax + by + c = 0$.



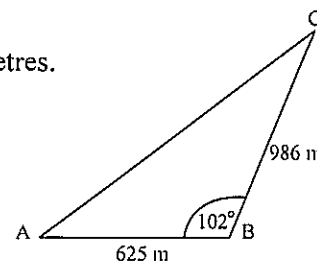
2. (a) Write down the gradient of the line $9x - 3y = -12$.
- (b) Find the gradient of the line which is perpendicular to the line $9x - 3y = -12$.
- (c) Find the equation of the line in slope intercept form which is perpendicular to $9x - 3y = -12$ and which passes through the point $(6, 7)$.
- (d) Find the coordinates of the point of intersection of these two lines.
3. $P(4, 1)$ and $Q(0, -5)$ are points on the coordinate plane.
- Determine the
 - coordinates of M , the midpoint of P and Q ;
 - distance between P and Q ;
 - gradient of the line drawn through P and Q ;

4. You were out flying a kite. You've given out 325 feet of string. If the angle of elevation from the ground up to the kite is 15° , how high is the kite?

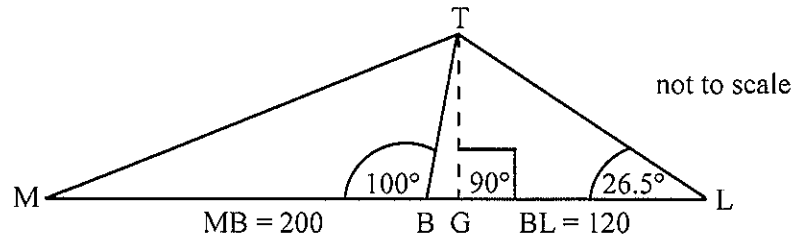
5. On a map three schools A, B and C are situated as shown in the diagram.

Schools A and B are 625 metres apart. Angle $\hat{A}BC = 102^\circ$ and $BC = 986$ metres.

- Find the distance between A and C.
- Find the size of angle $\hat{B}AC$.
- Find the area of the triangle created by ABC to 2 decimal places.

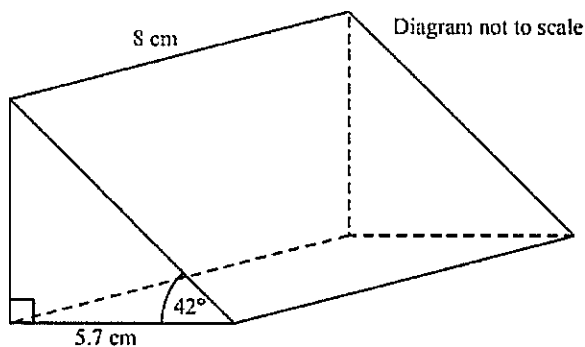


6. An old tower (BT) leans at 10° away from the vertical (represented by line TG). The base of the tower is at B so that $\widehat{MBT} = 100^\circ$. Leonardo stands at L on flat ground 120 m away from B in the direction of the lean. He measures the angle between the ground and the top of the tower T to be $\widehat{BLT} = 26.5^\circ$.

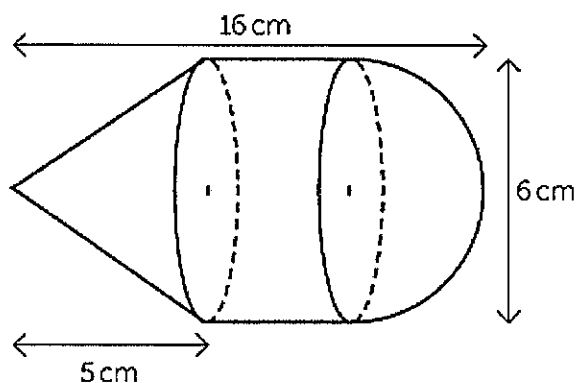


- (a) (i) Find the value of angle \widehat{BTL} .
- (ii) Use triangle BTL to calculate the sloping distance BT from the base, B to the top, T of the tower.
- (b) Calculate the vertical height TG of the top of the tower.

7. Find the volume and surface area of the prism below.

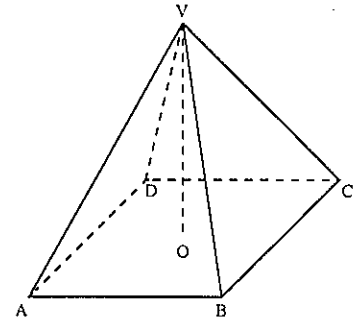


8. Calculate the surface area and volume of the object below.



9. ABCDV is a solid glass pyramid. The base of the pyramid is a square of side 3.2 cm. The vertical height is 2.8 cm. The vertex V is directly above the centre O of the base.

- Calculate the volume of the pyramid.
- The glass weighs 9.3 grams per cm^3 . Calculate the weight of the pyramid.
- Show that the length of the sloping edge VC of the pyramid is 3.6 cm.
- Calculate the total surface area of the pyramid.
- Find the angle that the front of the pyramid makes with the base.



10. A room is in the shape of a cuboid. Its floor measures 7.2 m by 9.6 m and its height is 3.5 m as shown below.

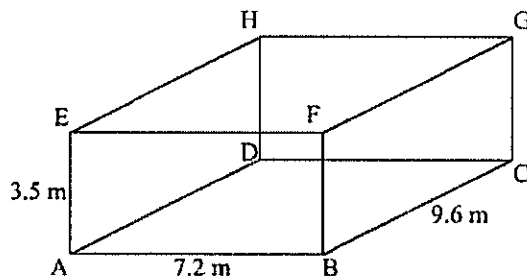


diagram not to scale

- Calculate the length of AC.
- Calculate the length of AG.
- Calculate the angle that AG makes with the floor.