Calculus Name Solutions ID: 1

Integral Review

Evaluate each indefinite integral. Note: None of these require u-substitution.

1) $\int 4\, dx$

$\boxed{4x + C}$

2) $\int (-2x^3 + 15x^2 + 2x)\, dx$

$-\frac{2}{4}x^4 + \frac{15}{3}x^3 + \frac{2}{2}x^2 + C$

$\boxed{-\frac{1}{2}x^4 + 5x^3 + x^2 + C}$

3) $\int \left(\frac{8}{x^3} - \frac{12}{x^5}\right) dx = \int (8x^{-3} - 12x^{-5})dx$

$\frac{8x^{-2}}{-2} - \frac{12}{-4}x^{-4} = -4x^{-2} + 3x^{-4} + C$

$\boxed{\frac{-4}{x^2} + \frac{3}{x^4} + C}$

4) $\int \sqrt[3]{x^2}\, dx = \int x^{2/3}\, dx$

$\frac{3}{5} \cdot x^{5/3} + C$

or $\frac{3}{5}\sqrt[3]{x^5} + C$

5) $\int (15x^2 - 8\sqrt[3]{x} + 5\sqrt[4]{x})\, dx$

$\int (15x^2 - 8x^{1/3} + 5x^{1/4})\, dx$

$\frac{1}{3} \cdot 15x^3 - \frac{3}{4} \cdot 8x^{4/3} + \frac{4}{5} \cdot 5x^{5/4} + C$

$\boxed{5x^3 - 6x^{4/3} + 4x^{5/4} + C}$

6) $\int 6x(2x^4 + 1)\, dx = \int (12x^5 + 6x)\, dx$

$\frac{12}{6}x^6 + \frac{6}{2}x^2 + C$

$\boxed{2x^6 + 3x^2 + C}$

7) $\int 3e^x\, dx$

$\boxed{3e^x + C}$

8) $\int \frac{5}{x}\, dx = \int 5 \cdot \frac{1}{x}\, dx$

$\boxed{5 \ln|x| + C}$

9) $\int 5\sin x\, dx = 5\int \sin x\, dx$

$5(-\cos x) + C$

$\boxed{-5\cos x + C}$

10) $\int 4\sec^2 x\, dx = 4\int \sec^2 x\, dx$

$\boxed{4 \tan x + C}$

Evaluate each indefinite integral.

11) $\int 10x^4(2x^5+1)^5\,dx$

$u = 2x^5+1$

$du = 10x^4dx$

$\int u^5\,du$

$\frac{1}{6}u^6+C = \boxed{\frac{1}{6}(2x^5+1)^6+C}$

12) $\int \frac{12x^3}{(3x^4-5)^3}\,dx$

$u = 3x^4-5$

$du = 12x^3dx$

$\int \frac{1}{u^3}du = \int u^{-3}du$

$-\frac{1}{2}u^{-2}+C = \boxed{-\frac{1}{2(3x^4-5)^2}+C}$

13) $\int 5e^{5x}\cdot(e^{5x}+3)^3\,dx$

$u = e^{5x}+3$

$du = 5e^{5x}dx$

$\int u^3\,du$

$\frac{1}{4}u^4+C = \boxed{\frac{1}{4}(e^{5x}+3)^4+C}$

14) $\int 4x\sqrt[3]{2x^2-3}\,dx$

$u = 2x^2-3$

$du = 4x\,dx$

$\int u^{1/3}du$

$\frac{3}{4}u^{4/3}+C = \frac{3}{4}\sqrt[3]{(2x^2-3)^4}+C$

or $\boxed{\frac{3}{4}(2x^2-3)^{4/3}+C}$

15) $\int 48x^2(4x^3+3)^3\,dx$

$u = 4x^3+3$

$du = 12x^2dx$

$4du = 48x^2dx$

$\int u^3\cdot 4du$

$4\int u^3du = \frac{1}{4}\cdot 4u^4+C$

$u^4+C = \boxed{(4x^3+3)^4+C}$

16) $\int 30x^2(\cancel{5}\,20x^3-4)^3\,dx$

$u = 20x^3-4$

$du = 60x^2dx$

$\frac{1}{2}du = 30x^2dx$

$\int u^3\cdot\frac{1}{2}du$

$\frac{1}{2}\int u^3\,du$

$\frac{1}{4}\cdot\frac{1}{2}u^4+C = \boxed{\frac{1}{8}(20x^3-4)^4+C}$

17) $\int 5x^4e^{x^5-5}\,dx$

$u = x^5-5$

$du = 5x^4dx$

$\int e^u\,du$

$e^u+C = \boxed{e^{x^5-5}+C}$

18) $\int \frac{12x^2}{4x^3+3}\,dx$

$u = 4x^3+3$

$du = 12x^2dx$

$\int \frac{1}{u}du$

$\ln|u|+C = \boxed{\ln|4x^3+3|+C}$

19) $\int 20x^3\cos(5x^4-2)\,dx$

$u = 5x^4-2$

$du = 20x^3dx$

$\int \cos u\,du$

$\sin u+C$

$\boxed{\sin(5x^4-2)+C}$

20) $\int 4x^3\sin(2x^4+3)\,dx$

$u = 2x^4+3$

$du = 8x^3dx$

$\frac{1}{2}du = 4x^3dx$

$\int \sin u\cdot\frac{1}{2}du$

$\frac{1}{2}\int \sin u\,du$

$\frac{1}{2}(-\cos u)+C = \boxed{-\frac{1}{2}\cos(2x^4+3)+C}$

Evaluate each definite integral.

21) $\int_{-1}^{3} (x^3 - 3x^2 + 4)\, dx$

$\frac{1}{4}x^4 - \frac{1}{3}\cdot 3x^3 + 4x \Big]_{-1}^{3}$

$\frac{1}{4}x^4 - x^3 + 4x \Big]_{-1}^{3}$

$\left[\frac{1}{4}(3)^4 - (3)^3 + 4(3)\right] - \left[\frac{1}{4}(-1)^4 - (-1)^3 + 4(-1)\right]$

$5.25 - (-2.75) = \boxed{8}$

22) $\int_{-3}^{-1} -\frac{3}{x^2}\, dx = -3\int_{-3}^{-1} \frac{1}{x^2} dx = -3\int x^{-2} dx$

$-3\cdot(-1)\, x^{-1} \Big]_{-3}^{-1}$

$\frac{3}{x}\Big]_{-3}^{-1} = \frac{3}{-1} - \frac{3}{-3}$

$-3 + 1 = \boxed{-2}$

23) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -2\cos x\, dx$

$-2(\sin x)\Big]_{-\pi/2}^{\pi/2}$

$-2\sin x \Big]_{-\pi/2}^{\pi/2}$

$-2\sin(\pi/2) - (-2\sin(-\pi/2))$

$-2(1) + 2(-1) = \boxed{-4}$

24) $\int_{2}^{4} \frac{5}{x}\, dx = 5\int_2^4 \frac{1}{x} dx$

$5\ln|x| \Big]_2^4$

$5(\ln 4 - \ln 2) = 5\ln\frac{4}{2} = \boxed{5\ln 2 \approx 3.466}$

Express each definite integral in terms of u, but do not evaluate.

25) $\int_{-2}^{1} \frac{4x}{(2x^2 + 1)^2}\, dx;\ u = 2x^2 + 1$ $du = 4x\,dx$

$u = 2(1)^2 + 1 = 3$

$u = 2(-2)^2 + 1 = 9$

$\boxed{\int_9^3 \frac{1}{u^2}\, du}$

26) $\int_{0}^{1} -6x(x^2 + 2)^2\, dx;\ u = x^2 + 2$ $du = 2x\,dx$

$-3du = -6x\,dx$

$u = (1)^2 + 2 = 3$

$u = (0)^2 + 2 = 2$

$\int_2^3 u^2\,(-3du)$

$\boxed{\int_2^3 -3u^2\, du}$

Evaluate each definite integral.

27) $\int_{-3}^{-1} \frac{4x}{(2x^2 + 2)^2}\, dx$

$u = 2x^2 + 2$

$du = 4x\,dx$

$u = 2(-1)^2 + 2 = 4$

$u = 2(-3)^2 + 2 = 20$

$\int_{20}^{4} u^{-2}\, du$

$-1u^{-1}\Big]_{20}^{4}$

$\frac{-1}{4} - \left(\frac{-1}{20}\right) = \frac{-1}{4} + \frac{1}{20} =$

$\frac{-5}{20} + \frac{1}{20} = \frac{-4}{20} = \boxed{-1/5}$

28) $\int_{-1}^{0} 12x(3x^2 - 4)^2\, dx$

$u = 3x^2 - 4$

$du = 6x\,dx$

$2du = 12x\,dx$

$u = 3(0)^2 - 4 = -4$

$u = 3(-1)^2 - 4 = -1$

$\int_{-1}^{-4} u^2 \cdot 2du$

$2\int_{-1}^{-4} u^2\, du = \frac{2}{3}u^3\Big]_{-1}^{-4}$

$\frac{2}{3}\left[(-4)^3 - (-1)^3\right] = \frac{2}{3}\left[-64 + 1\right]$

$= \boxed{-42}$

For each problem, find the area under the curve over the given interval.

29) $y = x^2 - 4x + 5$; $[0, 3]$

$$\boxed{\int_0^3 (x^2 - 4x + 5)\,dx}$$

Let's use the calc to evaluate ...

$$\boxed{6}$$

30) $y = \frac{1}{x^2}$; $[-2, -1]$

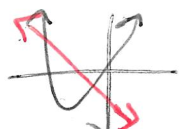
$$\boxed{\int_{-2}^{-1} \left(\frac{1}{x^2}\right)dx}$$

Let's use the calc to evaluate

$$\boxed{1/2}$$

For each problem, find the area of the region enclosed by the curves.

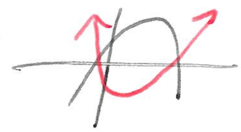
31) $y = 2x^2 + 16x + 29$, $\cancel{y = x - 1}$ $y = -2x - 7$,
$x = -6$, $x = -3$



$$\int_{-6}^{-3} \left(-2x - 7 - (2x^2 + 16x + 29)\right)dx$$

$$\boxed{9}$$

32) $y = -x^2 + 4x + 6$, $y = x^2 - 4x - 5$,
$x = -1$, $x = 5$



$$\int_{-1}^{5} \left(-x^2 + 4x + 6 - (x^2 - 4x - 5)\right)dx$$

$$\boxed{78}$$

33) $y = -x^2 + 2x$, $y = 2x^3 - 4x$,
$x = -2$, $x = 1.5$

$$\int_{-2}^{0} \left(2x^3 - 4x - (-x^2 + 2x)\right)dx +$$

$$\int_{0}^{1.5} \left(-x^2 + 2x - (2x^3 - 4x)\right)dx$$

$$\approx \boxed{9.76}$$

Formula

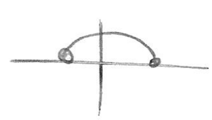
$$\boxed{\int \pi y^2}$$

For each problem, find the volume of the solid formed when the given function is rotated 360 degrees on the given integral.

34) $y = \sqrt{36 - x^2}$, the x-axis

intercepts at ± 6

$$\int_{-6}^{6} \pi \left(\sqrt{36 - x^2}\right)^2 dx$$



$$\boxed{\pi \int_{-6}^{6} (36 - x^2)\,dx}$$

Let's use the calc

$$\boxed{288\pi \approx 904.78 \text{ units}^3}$$

35) $y = (2 + x)^2$, from $x = 0$ to $x = 1$

$$\int_0^1 \pi \left((2 + x)^2\right)^2 dx$$

$$\boxed{\pi \int_0^1 (2 + x)^4\,dx}$$

Let's use calc ...

$$\boxed{42.2\pi \approx 132.58 \text{ units}^3}$$