Exercise 13F Solutions

1 (a) $\sin(2x) = \cos x$

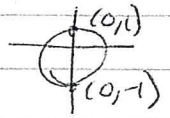
$2\sin x\cos x = \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\quad 2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$



90°

Ref

30°, 150°

(b) $\sin(2x) = \cos(2x)$

~~$2\sin x\cos x$~~

$\frac{\sin(2x)}{\cos(2x)} = 1$

$\tan(2x) = 1$

$\tan\theta = 1$

Ref 45°

$\theta = 45°, 225°$

$2x = 45°, 225°$

$x = 22.5°; 112.5°$

(c) $(\sin x + \cos x)^2 = 0$

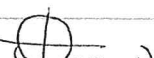
$\sin^2 x + 2\sin x\cos x + \cos^2 x = 0$

$2\sin x\cos x + \sin^2 x + \cos^2 x = 0$

$2\sin x\cos x + 1 = 0$

$\sin(2x) + 1 = 0$

$\sin(2x) = -1 \rightarrow \sin\theta = -1$



$2x = 270°$

$x = 135°$

2 (a) $2\sin x\cos x = \sqrt{3}/2$

$\sin(2x) = \sqrt{3}/2$

$\sin\theta = \sqrt{3}/2$

Ref 60°

$\theta = 60, 120, -240, -300$

$2x = 60, 120, -240, -300$

$x = 30°, 60°, -120°, -150°$

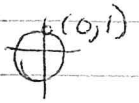
(b) $\sin x(1 - \sin x) = \cos^2 x$

$\sin x - \sin^2 x = \cos^2 x$

$\sin x = \sin^2 x + \cos^2 x$

$\sin x = 1$

$x = 90°$



(c) $\cos^2 x = \frac{1}{2} + \sin^2 x$

$\cos^2 x - \sin^2 x = \frac{1}{2}$

$\cos(2x) = \frac{1}{2}$

Ref 60°

$\theta = 60, 300, -60, -300$

$2x = 60, 300, -60, -300$

$x = 30, 150, -30, -150°$

(d) $\cos(2x) = \sin x$

$1 - 2\sin^2 x = \sin x$

$0 = 2\sin^2 x + \sin x - 1$

$0 = (2\sin x - 1)(\sin x + 1)$

$\sin x = 1/2 \quad \sin x$

$\sin x = 1/2$ Ref $30°$ — S, A, T, C

$30°, 150°$

$\sin x = -1$

$(0, -1)$

~~$270°$~~, $-90°$

3 (a) $\tan x = \sin x$

$\frac{\sin x}{\cos x} = \sin x$

$\sin x = \sin x \cos x$

$0 = \sin x \cos x - \sin x$

$0 = \sin x(\cos x - 1)$

$\sin x = 0$

$0, \pi$

$\cos x = 1$

0

(b) $2\cos^2 x - 1 = 1/\sqrt{2}$

$\cos(2x) = \sqrt{2}/2$

$\cos\theta = \sqrt{2}/2$

Ref $\pi/4$ — S, A, T, C

$\theta = \pi/4, 7\pi/4$

$2x = \pi/4, 7\pi/4$

$x = \pi/8, 7\pi/8$

4 (b) $\sin x - 1 = \cos^2 x$

$\sin x - 1 = 1 - \sin^2 x$

$\sin^2 x + \sin x - 2 = 0$

$(\sin x + 2)(\sin x - 1) = 0$

$\sin x = -2$

no solution

$\sin x = 1$

$\pi/2$

5 (a) $(\sin x + \cos x)^2 = 1 + \sin(2x)$

$\sin^2 x + 2\sin x \cos x + \cos^2 x = 1 + \sin(2x)$

$\sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + \sin(2x)$

$1 + 2\sin x \cos x = 1 + 2\sin x \cos x$ ✓

(b) $1/\cos\theta = \sin\theta \tan\theta + \cos\theta$

$1/\cos\theta = \sin\theta \frac{\sin\theta}{\cos\theta} + \cos\theta$

$1/\cos\theta = \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta}$

$1/\cos\theta = \frac{1}{\cos\theta}$ ✓

(c) $\frac{1-\cos^2(2x)}{2\sin x\cos x} = 2\sin x\cos x$

$\frac{\sin^2(2x)}{\sin(2x)} = \sin(2x)$

$\sin(2x) = \sin(2x)$ ✓

f/i: $\sin^2 x + \cos^2 x = 1$

$\sin^2(2x) + \cos^2(2x) = 1$

$\sin^2(2x) = 1 - \cos^2(2x)$

(d) $\cos\theta + \sin\theta = \frac{1-2\sin^2\theta}{\cos\theta - \sin\theta}$

$\cos\theta + \sin\theta = \frac{\cos(2x)}{\cos\theta - \sin\theta}$

$\cos\theta + \sin\theta = \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta}$

$\cos\theta + \sin\theta = \frac{(\cos^2\theta + \sin\theta)(\cos\theta - \sin\theta)}{\cos\theta - \sin\theta}$

$\cos\theta + \sin\theta = \cos\theta + \sin\theta$ ✓

(e) $\cos^4 x - \sin^4 x = \cos(2x)$

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) =$

$1(\cos^2 x - \sin^2 x) = \cos 2x$

$\cos 2x = \cos 2x$ ✓

(6) $\sin 2x = 2\sin x\cos x$

$\sin 2(2x) = 2\sin 2x\cos 2x$

$\sin 3(2x) = 2\sin 3x\cos 3x$

$\sin 6x = 2\sin 3x\cos 3x$

$k = 6$

(7) $\cos 4x = \cos 2(2x)$

$\cos 2x = 1 - 2\sin^2 x$

$\cos 2(2x) = 1 - 2\sin^2(2x)$

$\cos 4x = 1 - b\sin^2 x\cos^2 x$

$1 - 2\sin^2(2x) = 1 - b\sin^2 x\cos^2 x$

$1 - 2(\sin(2x))^2 = 1 - b\sin^2 x\cos^2 x$

$1 - 2(2\sin x\cos x)^2 = 1 - b\sin^2 x\cos^2 x$

$1 - 8\sin^2 x\cos^2 x = 1 - b\sin^2 x\cos^2 x$

$b = 8$