Hmwk J solutions

1) $\int_{-1}^{1} \frac{1}{t+2}\,dt$ $\quad u = t+2$ $\quad du = 1dt$ $\quad$ if $t=-1$ $u=-1+2=1$ $\quad$ if $t=1$ $u=1+2=3$

$$\int_1^3 \frac{1}{u}\,du = \ln|u|\Big]_1^3 = \ln 3 - \ln 1 = \ln 3 - 0 = \boxed{\ln 3}$$

2) $\int_3^4 e^{-x+1}\,dx$ $\quad u = -x+1$ $\quad du = -1dx$ $\quad -1du = dx$ $\quad$ if $x=3$ $u=-3+1=-2$ $\quad$ if $x=4$ $u=-4+1=-3$

$$\int_{-2}^{-3} e^{u}(-1du)$$

$$-1\int_{-2}^{-3} e^{u}\,du = -e^{u}\Big]_{-2}^{-3} = -e^{-3}-(-e^{-2}) = -e^{-3}+e^{-2}$$

$$\boxed{\frac{-1}{e^3}+\frac{1}{e^2}}$$

3) $\int_{-1}^{2} (-2x+1)^3\,dx$ $\quad u = -2x+1$ $\quad du = -2dx$ $\quad -\frac{1}{2}du = dx$ $\quad$ if $x=-1$ $u=-2(-1)+1=3$ $\quad x=2$ $u=-2(2)+1=-3$

$$\int_3^{-3} (u)^3\left(-\tfrac{1}{2}du\right) = -\frac{1}{2}\int_3^{-3} u^3 = \frac{-1}{2}\cdot\frac{1}{4}u^4\Big]_3^{-3}$$

$$-\frac{1}{8}u^4\Big]_3^{-3} = \frac{-1}{8}\left[(-3)^4-(3)^4\right] = \frac{-1}{8}(81-81) = \frac{-1}{8}(0) = \boxed{0}$$

4) $\int_{-1}^{1}(e^x+e^{-x})dx = \int_{-1}^{1} e^x dx + \int_{1}^{1} e^{-x} dx$

$e^x\Big]_{-1}^{1} = e^1 - (e^{-1})$

$e - \frac{1}{e}$

$\int_{1}^{-1} e^u(-1du)$

$u = -x$
$du = -1dx$
$-1du = dx$

if $x=-1$ $u=1$
if $x=1$ $u=-1$

$-1\int_{1}^{-1} e^u du$

$-1e^u\Big]_{1}^{-1} = -1e^{-1} - (-1e^1) = \frac{-1}{e} + e$

$(e - \frac{1}{e}) + (\frac{-1}{e} + e) = \boxed{2e - \frac{2}{e}}$ or $\boxed{2\left(e - \frac{1}{e}\right)}$

5) $\int_{0}^{2}(6x+4)^{\frac{1}{2}} dx$

$u = 6x+4$
$du = 6dx$
$\frac{1}{6}du = dx$

if $x=0$ $u=6(0)+4=4$
$x=2$ $u=6(2)+4$ 16

$\int_{4}^{16} u^{\frac{1}{2}} \cdot \frac{1}{6}du = \frac{1}{6}\int_{4}^{16} u^{\frac{1}{2}}du = \frac{1}{6}\cdot\frac{2}{3}u^{3/2}\Big]_{4}^{16} = \frac{1}{9}u^{3/2}\Big]_{4}^{16}$

$\frac{1}{9}\left((16^{\frac{1}{2}})^3 - (4^{\frac{1}{2}})^3\right) = \frac{1}{9}(4^3 - 2^3) = \frac{1}{9}(64-8) = \frac{1}{9}(56)$

$\boxed{\frac{56}{9}}$

6) $\int_1^2 (x^2+x)^3 (2x+1)\,dx$

$u = x^2+x$ if $x=1$ $u=2$

$du = (2x+1)\,dx$ $x=2$ $u=6$

$\int_2^6 u^3\,du = \frac{1}{4}u^4\Big]_2^6 \quad \frac{1}{4}(6^4-2^4) = \frac{1}{4}(1296-16) = \frac{1}{4}(1280)$

$= \boxed{320}$

7) $\int_3^4 \frac{8t-6}{2t^2-3t-2}\,dt$

$u = 2t^2-3t-2$ if $x=3$ $u=7$

$du = (4t-3)\,dt$ $x=4$ $u=18$

$2du = (8t-6)\,dt$

$\int_3^4 \frac{1}{2t^2-3t-2} \cdot (8t-6)\,dt$

$\int_7^{18} \frac{1}{u} \cdot 2du = 2\int_7^{18} \frac{1}{u}\,du \quad 2\ln|u|\Big]_7^{18} \quad 2\left[\ln 18 - \ln 7\right]$

$\boxed{2\ln\frac{18}{7}}$

8) $\int_0^1 4xe^{x^2+3}\,dx$

$u = x^2+3$ if $x=0$ $u=3$

$du = 2x\,dx$ $x=1$ $u=4$

$2du = 4x\,dx$

$\int_3^4 e^u \cdot 2du = 2\int_3^4 e^u\,du = 2e^u\Big]_3^4 = \boxed{2(e^4-e^3)}$

9) ⓐ Since the graph crosses the x-axis, find the intercepts by setting the equation equal to 0.

$-2x^2(x-2)=0$

$-2x^2=0 \qquad x-2=0$

$x=0 \qquad x=2$

ⓐ $\boxed{\int_0^2 -2x^2(x-2)\,dx}$

ⓑ $\int_0^2 -2x^2(x-2)\,dx \qquad \int_0^2 (-2x^3+4x^2)\,dx$

$-2\cdot\frac{1}{4}x^4 + 4\cdot\frac{1}{3}x^3\Big]_0^2 = -\frac{1}{2}x^4 + \frac{4}{3}x^3\Big]_0^2$

$\left(-\frac{1}{2}(2)^4 + \frac{4}{3}(2)^3\right) - \left(-\frac{1}{2}(0)^4 + \frac{4}{3}(0)^3\right)$

$\left(-\frac{1}{2}\cdot 16 + \frac{4}{3}\cdot 8\right) - 0 = -8 + \frac{32}{3} = -\frac{24}{3} + \frac{32}{3} = \boxed{\frac{8}{3}}$

10) $\int_2^k \frac{1}{x-1}\,dx \qquad u = x-1 \qquad \int_1^{k-1} \frac{1}{u}\,du$

$du = dx$

Area $= \ln 4$

if $x=2$ $u=1$

if $x=k$ $u=k-1$

$\ln|u|\Big]_1^{k-1}$

$\ln|k-1| - \ln 1 = \underset{\text{Area}}{\ln 4}$

$\ln|k-1| - 0 = \ln 4$

$\ln|k-1| = \ln 4$

$|k-1| = 4$

$k=5$

note: $k \neq -3$ because on the graph k is to the right of $x=2$