Hmwk 8

1) $\int(2\cos x + 3\sin x)dx = \boxed{2\sin x - 3\cos x + C}$

2) $\int(x^2 + \cos(\frac{1}{3}x))dx$

$\int x^2 + \int\cos(\frac{1}{3}x)dx$

$u = \frac{1}{3}x$
$du = \frac{1}{3}dx$
$3du = dx$

$\int x^2 + \int\cos u \cdot 3du$

$\int x^2 + 3\int\cos u\,du$

$\frac{1}{3}x^3 + 3\sin u + C = \boxed{\frac{1}{3}x^3 + 3\sin(\frac{1}{3}x) + C}$

3) $\int\pi\sin(\pi x)dx$

$u = \pi x$
$du = \pi dx$

$\int\sin(\pi x)\pi dx$
$\int\sin u\,du$
$-\cos u + C = \boxed{-\cos(\pi x) + C}$

4) $\int\sin(2x+3)dx$

$u = 2x+3$
$du = 2dx$
$\frac{1}{2}du = dx$

$\int\sin u \cdot \frac{1}{2}du$

$\frac{1}{2}\int\sin u\,du = \frac{1}{2}(-\cos u) + C = \boxed{-\frac{1}{2}\cos(2x+3) + C}$

5) $\int 20x^3\cos(5x^4)dx$

$u = 5x^4$
$du = 20x^3dx$

$\int\cos(5x^4)20x^3dx$
$\int\cos u \cdot du$
$\sin u + C = \boxed{\sin(5x^4) + C}$

6) $\int (2x-1)\cos(4x^2-4x)\,dx$

$u = 4x^2-4x$
$du = (8x-4)\,dx$
$\frac{1}{4}du = (2x-1)\,dx$

$\int \cos(4x^2-4x)(2x-1)\,dx$

$\int \cos u \cdot \frac{1}{4}du$

$1/4 \int \cos u\,du = \frac{1}{4}\sin u + C = \boxed{\frac{1}{4}\sin(4x^2-4x)+C}$

7) $\int \frac{e^{\tan(3x)}}{\cos^2(3x)}\,dx = \int e^{\tan(3x)} \cdot \frac{1}{\cos^2(3x)}\,dx$

$u = \tan 3x$
$du = 3 \cdot \frac{1}{\cos^2(3x)}\,dx$
$\frac{1}{3}du = \frac{1}{\cos^2(3x)}\,dx$

$\int e^u \cdot \frac{1}{3}du = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{\tan 3x} + C}$

8) $\int \frac{\cos(\ln x)}{x}\,dx = \int \cos(\ln x)\frac{1}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}dx$

$\int \cos u \cdot du = \sin u + C = \boxed{\sin(\ln x) + C}$

9) $\int \cos x \sin^2 x\,dx = \int \cos x(\sin x)^2\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\sin x)^3 + C$

or $\boxed{\frac{1}{3}\sin^3 x + C}$

10) $\int \frac{\sin x}{\cos x}\,dx$ for $\cos x > 0$ $\quad \int \sin x \cdot \frac{1}{\cos x}\,dx$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$\int \frac{1}{u}(-du) = -\int \frac{1}{u}\,du = -\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

11) (a) $f(x) = e^{\sin x}\cos x$
$f'(x) = \ ?$ (CR + PR)

$f'(x) = e^{\sin x} \cdot (-\sin x) + \cos x \cdot e^{\sin x} \cdot \cos x$

$\boxed{-e^{\sin x} \cdot \sin x + e^{\sin x} \cdot \cos^2 x}$

(b) $\int e^{\sin x} \cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$

$\int e^u\,du = e^u + C = \boxed{e^{\sin x} + C}$

12) (a) $f(x) = \ln(\cos x)$
$f'(x) = \ ?$ (CR)

$f'(x) = \frac{1}{\cos x} \cdot (-\sin x)$

$= \frac{-\sin x}{\cos x}$

$= -\tan(x)$ ✓

(b) $\int \tan x \cdot \ln(\cos x)\,dx$

"Hence" means use what you did in the last part to help

$u = \ln(\cos x)$
$du = -\tan x\,dx \quad -du = \tan x\,dx$

$\int u(-du) = -\int u\,du = -\frac{1}{2}u^2 + C$

$\boxed{-\frac{1}{2}(\ln(\cos x))^2 + C}$