

Hmwk 12L Solutions

1) a) Ship - Buoy
$$\begin{pmatrix} 60 \\ 30 \end{pmatrix} - \begin{pmatrix} 45 \\ 20 \end{pmatrix} = \begin{pmatrix} 15 \\ 10 \end{pmatrix} \quad \begin{array}{l} 15 \text{ km east} \\ 10 \text{ km north} \end{array}$$

b)
$$\sqrt{(15)^2 + (10)^2} = \sqrt{325} = \sqrt{25 \cdot 13} = 5\sqrt{13} \text{ km}$$

2) a)
$$\frac{1}{4} \begin{pmatrix} 20 \\ -8 \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \text{ m/s}$$

b)
$$r = \begin{pmatrix} 20 \\ -8 \end{pmatrix} + t \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

position velocity

$$r = \begin{pmatrix} 20 \\ -8 \end{pmatrix} + (6) \begin{pmatrix} 5 \\ -2 \end{pmatrix} \rightarrow \begin{pmatrix} 20 \\ -8 \end{pmatrix} + \begin{pmatrix} 30 \\ -12 \end{pmatrix} = \begin{pmatrix} 50 \\ -20 \end{pmatrix} \text{ m}$$

c) speed = magnitude of velocity =
$$\sqrt{(12)^2 + (-5)^2} = \sqrt{169} = 13 \text{ m/s}$$

d)
$$s = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + t \begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

position velocity

$$s = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + (3) \begin{pmatrix} 12 \\ -5 \end{pmatrix} \rightarrow \begin{pmatrix} 40 \\ -16 \end{pmatrix} \quad \sqrt{40^2 + (-16)^2} \approx 43.1 \text{ m/s}$$

e) $r = s$?

$$\begin{pmatrix} 20 \\ -8 \end{pmatrix} + t \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + s \begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

$$20 + 5t = 4 + 12s$$

$$-8 - 2t = -1 - 5s \rightarrow t = -7/2 + 5/2s$$

substitute

$$20 + 5(-7/2 + 5/2s) = 4 + 12s$$

$$s = 3$$

plug back in and both t's = 4 so they collide

$$3) \textcircled{a} r_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad r_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \end{pmatrix} s$$

$$r_1 = r_2 \quad \begin{pmatrix} 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + s \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad \begin{array}{l} 3+4t = 4+3s \\ 3+3t = 3+3s \end{array}$$

$$\begin{array}{l} \swarrow t=s \\ \text{substitution} \\ 3+4t = 4+3t \\ t=1 \end{array}$$

collide in 1 hr.
4 PM

$$\textcircled{b} r_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + (1) \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \text{or} \quad r_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix} + (1) \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 6 \end{pmatrix} \quad \text{or} \quad 7i + 6j$$

$$4) \textcircled{a} \sqrt{(1)^2 + (-1)^2 + (4)^2} = \sqrt{18} = 4.2 \text{ m/s}$$

$$\sqrt{(2)^2 + (1)^2 + (9)^2} = \sqrt{86} = 9.3 \text{ m/s}$$

$$\textcircled{b} \text{ set } \begin{array}{l} 11+t = 1+2s \\ r_x = r_y \quad 3-t = -7+s \\ -3+4t = -2+9s \end{array}$$

$$\text{elimination } \begin{array}{l} 14 = -6+3s \\ s = 20/3 \end{array}$$

$$\text{plug } s \text{ into all 3 eqs}$$

$$t = 10/3 \quad t = 10/3 \quad t = 61/4$$

Since the t's are not all equal \rightarrow does not collide

$$\textcircled{c} \begin{pmatrix} 11 \\ 3 \\ -3 \end{pmatrix} + 10 \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 21 \\ -7 \\ 37 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -7 \\ -2 \end{pmatrix} + 10 \begin{pmatrix} 2 \\ 1 \\ 9 \end{pmatrix} = \begin{pmatrix} 21 \\ 3 \\ 88 \end{pmatrix}$$

$$\begin{pmatrix} 21 \\ -7 \\ 37 \end{pmatrix} - \begin{pmatrix} 21 \\ 3 \\ 88 \end{pmatrix} = \begin{pmatrix} 0 \\ -10 \\ -51 \end{pmatrix}$$

$$\sqrt{0^2 + (-10)^2 + (-51)^2} = \sqrt{2701} = \boxed{52.0 \text{ m}}$$