Exercise 12D solutions 3 & 4

(3) $\overrightarrow{P_1P_2} = P_2 - P_1 = -3i - 1j + 0k$

$\overrightarrow{P_1P_3} = P_3 - P_1 = -6i - 2j + 0k$

collinear since $2\overrightarrow{P_1P_2} = \overrightarrow{P_1P_3}$ and they share P_1

$\overrightarrow{P_1P_4} = P_4 - P_1 = (2, y, z) - (1, 2, 4) = \begin{pmatrix} 1 \\ y-2 \\ z-4 \end{pmatrix}$

$\overrightarrow{P_1P_4} = k\overrightarrow{P_1P_2}$ to be collinear

$$\begin{pmatrix} 1 \\ y-2 \\ z-4 \end{pmatrix} = k\begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix}$$

$1 = -3k$

$y - 2 = -k$

$z - 4 = 0$

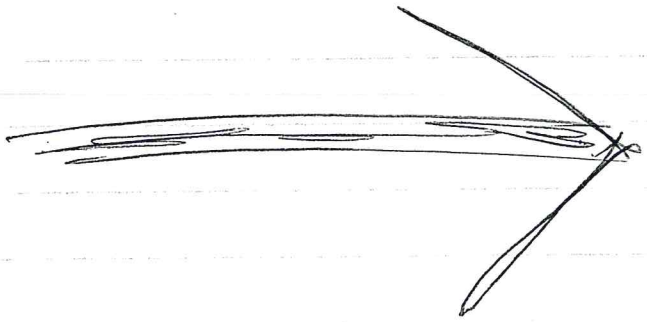
so $1 = -3k$ $\quad k = -1/3$

$z - 4 = 0$ $\quad \boxed{z = 4}$

$y - 2 = -k$

$y - 2 = 1/3$

$\boxed{y = 7/3}$



(4) $A: (3, 4)$ $B: (x, 0)$ $C: (1, -2)$

$\overrightarrow{AB}: \begin{pmatrix} x-3 \\ -4 \end{pmatrix}$ $\overrightarrow{AC}: \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

If collinear

$\overrightarrow{AB} = k\overrightarrow{AC}$

$\begin{pmatrix} x-3 \\ -4 \end{pmatrix} = k\begin{pmatrix} -2 \\ -6 \end{pmatrix}$ $\rightarrow$ $x-3 = -2k$, $-4 = -6k$

$k = 2/3 \rightarrow x-3 = -2(2/3)$

$x-3 = -4/3$

$x = 5/3$

$\overrightarrow{AB}: \begin{pmatrix} 5/3-3 \\ -4 \end{pmatrix} = \begin{pmatrix} -4/3 \\ -4 \end{pmatrix}$

$\overrightarrow{BC}: \begin{pmatrix} 1-5/3 \\ -2 \end{pmatrix} = \begin{pmatrix} -2/3 \\ -2 \end{pmatrix}$

ratio $AB:BC$ $\begin{pmatrix} -4/3 \\ -4 \end{pmatrix} : \begin{pmatrix} -2/3 \\ -2 \end{pmatrix}$

2 : 1