

Derivative Review

For each problem, find the following values

- 1) Find the derivative of

$$f(x) = -x^3 + 2x^2 + 4 \text{ at } x = 2$$

$$f'(x) = -3x^2 + 4x$$

$$f'(2) = -4$$

- 2) Find the slope of

$$f(x) = -x^2 - 6x - 5 \text{ at } x = 0$$

$$f'(x) = -2x - 6$$

$$f'(0) = -6$$

For each problem, find the equation of the line tangent or normal to the function at the given point. Your answer should be in slope-intercept form.

- 3) Tangent line of

$$f(x) = -x^3 + 2x^2 - 3 \text{ at } (2, -3)$$

$$f'(x) = -3x^2 + 4x$$

$$m = f'(2) = -4$$

$$y - (-3) = -4(x - 2)$$

$$y = -4x + 5$$

- 4) Normal line of

$$f(x) = -x^2 - 4x - 3 \text{ at } (-3, 0)$$

$$f'(x) = -2x - 4$$

$$m = f'(-3) = 2$$

$$\text{normal slope} = -\frac{1}{2}$$

$$y - 0 = -\frac{1}{2}(x - (-3))$$

$$y = -\frac{1}{2}x - \frac{3}{2}$$

For each problem, find the following

- 5) Find the point(s) where the slope of

$$f(x) = x^3 - 3x^2 - 1 \text{ equals } 0.$$

$$f'(x) = 3x^2 - 6x$$

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

$$x = 0 \text{ and } x = 2$$

$$f(0) = -1 \text{ and } f(2) = -5$$

$$(0, -1), (2, -5)$$

- 6) Find the point(s) where the slope of

$$f(x) = 2x^2 - 3x + 4 \text{ equals } 5.$$

$$f'(x) = 4x - 3$$

$$4x - 3 = 5$$

$$x = 2$$

$$f(2) = 6$$

$$(2, 6)$$

Differentiate each function with respect to x .

7) $f(x) = 3x^5 + 2x^2 + 1$

$$f'(x) = 15x^4 + 4x$$

8) $f(x) = -3x^4 + 4x + 10\sqrt{x}$

$$f'(x) = -12x^3 + 4 + \frac{5}{x^{\frac{1}{2}}}$$

$$9) f(x) = 2x^5 + 3x^2 + 6\sqrt[3]{x}$$

$$f'(x) = 10x^4 + 6x + \frac{2}{x^{\frac{2}{3}}}$$

$$10) f(x) = 3x + \frac{4}{x} - \frac{3}{x^2}$$

$$f'(x) = 3 - \frac{4}{x^2} + \frac{6}{x^3}$$

For each problem, find the indicated derivative with respect to x .

$$11) f(x) = 2x^5 - 5x^4 - x^2 + 3x \quad \text{Find } f''$$

$$f'(x) = 10x^4 - 20x^3 - 2x + 3$$

$$f''(x) = 40x^3 - 60x^2 - 2$$

$$12) f(x) = \frac{3}{x^5} + 4e^x \quad \text{Find } f''$$

$$f'(x) = -\frac{15}{x^6} + 4e^x$$

$$f''(x) = \frac{90}{x^7} + 4e^x$$

Differentiate each function with respect to x .

$$13) f(x) = -3x^4(-5x^5 + 2)$$

$$f'(x) = -3x^4 \cdot -25x^4 + (-5x^5 + 2) \cdot -12x^3$$
$$= 135x^8 - 24x^3$$

$$14) f(x) = (2x^4 + 4)(-2x^4 + x^2 + 2)$$

$$f'(x) = (2x^4 + 4)(-8x^3 + 2x) + (-2x^4 + x^2 + 2) \cdot 8x^3$$
$$= -32x^7 + 12x^5 - 16x^3 + 8x$$

$$15) f(x) = \frac{4x^5}{3x^2 + 3}$$

$$\begin{aligned} f'(x) &= \frac{(3x^2 + 3) \cdot 20x^4 - 4x^5 \cdot 6x}{(3x^2 + 3)^2} \\ &= \frac{40x^6 + 60x^4}{(3x^2 + 3)^2} \end{aligned}$$

$$16) f(x) = \frac{5x^2 + 5}{2x^4 - 3}$$

$$\begin{aligned} f'(x) &= \frac{(2x^4 - 3) \cdot 10x - (5x^2 + 5) \cdot 8x^3}{(2x^4 - 3)^2} \\ &= \frac{-20x^5 - 40x^3 - 30x}{(2x^4 - 3)^2} \end{aligned}$$

$$17) f(x) = (-4x^4 + 3)^4$$

$$\begin{aligned} f'(x) &= 4(-4x^4 + 3)^3 \cdot -16x^3 \\ &= -64x^3(-4x^4 + 3)^3 \end{aligned}$$

$$18) f(x) = (3x + 2)^2$$

$$\begin{aligned} f'(x) &= 2(3x + 2) \cdot 3 \\ &= 6(3x + 2) \end{aligned}$$

$$19) f(x) = e^{2x^4}$$

$$f'(x) = e^{2x^4} \cdot 8x^3$$

$$20) f(x) = \ln 2x^5$$

$$\begin{aligned} f'(x) &= \frac{1}{2x^5} \cdot 10x^4 \\ &= \frac{5}{x} \end{aligned}$$

$$21) f(x) = \sin 5x^2$$

$$\begin{aligned} f'(x) &= \cos 5x^2 \cdot 10x \\ &= 10x \cos 5x^2 \end{aligned}$$

$$22) f(x) = 2 \tan 5x$$

$$\begin{aligned} f'(x) &= 2 \cdot \frac{1}{\cos^2 5x} \cdot 5 \\ &= \frac{10}{\cos^2 5x} \end{aligned}$$

$$23) f(x) = e^{3x} \cos 5x^3$$

$$\begin{aligned} f'(x) &= e^{3x} \cdot -1 \sin 5x^3 \cdot 15x^2 + \cos 5x^3 \cdot e^{3x} \cdot 3 \\ &= 3e^{3x}(-5x^2 \sin 5x^3 + \cos 5x^3) \end{aligned}$$