IB Math SL

Name Solutions ID: 1

Derivative Review

For each problem, find the following values

1) Find the derivative of
$f(x) = -x^3 + 2x^2 + 4$ at $x = 2$

$f'(x) = -3x^2 + 4x$

$f'(2) = -3(2)^2 + 4(2)$

$\boxed{-4}$

2) Find the slope of
$f(x) = -x^2 - 6x - 5$ at $x = 0$

$f'(x) = -2x - 6$

$f'(0) = -2(0) - 6$

$\boxed{-6}$

For each problem, find the equation of the line tangent or normal to the function at the given point. Your answer should be in slope-intercept form.

3) Tangent line of
$f(x) = -x^3 + 2x^2 - 3$ at $(2, -3)$

$f'(x) = -3x^2 + 4x$

$f'(2) = -3(2)^2 + 4(2) = -4 = m$

$y - y_1 = m(x - x_1)$

$y - (-3) = -4(x - 2)$

$y + 3 = -4x + 8$

$\boxed{y = -4x + 5}$

4) Normal line of
$f(x) = -x^2 - 4x - 3$ at $(-3, 0)$

$f'(x) = -2x - 4$

$f'(-3) = -2(-3) - 4 = 2$ $m = -\frac{1}{2}$

$y - 0 = -\frac{1}{2}(x - (-3))$

$\boxed{y = -\frac{1}{2}x - \frac{3}{2}}$

For each problem, find the following

5) Find the point(s) where the slope of
$f(x) = x^3 - 3x^2 - 1$ equals 0.

$f'(x) = 3x^2 - 6x$

$0 = 3x^2 - 6x$

$0 = 3x(x - 2)$

$x = 0, \ x = 2$

$f(0) = -1 \quad f(2) = -5$

$\boxed{(0, -1) \quad (2, -5)}$

6) Find the point(s) where the slope of
$f(x) = 2x^2 - 3x + 4$ equals 5.

$f'(x) = 4x - 3$

$5 = 4x - 3$

$8 = 4x$

$x = 2$

$f(2) = 6$

$\boxed{(2, 6)}$

Differentiate each function with respect to x.

7) $f(x) = 3x^5 + 2x^2 + 1$

$\boxed{f'(x) = 15x^4 + 4x}$

8) $f(x) = -3x^4 + 4x + 10\sqrt{x}$

$f(x) = -3x^4 + 4x + 10x^{1/2}$

$f'(x) = -12x^3 + 4 + 5x^{-1/2}$

$\boxed{f'(x) = -12x^3 + 4 + \frac{5}{\sqrt{x}}}$

9) $f(x) = 2x^5 + 3x^2 + 6\sqrt[3]{x}$

$f(x) = 2x^5 + 3x^2 + 6x^{1/3}$

$f'(x) = 10x^4 + 6x + 2x^{-2/3}$

$\boxed{f'(x) = 10x^4 + 6x + \dfrac{2}{x^{2/3}} \quad \text{or} \quad 10x^4 + 6x + \dfrac{2}{\sqrt[3]{x^2}}}$

10) $f(x) = 3x + \dfrac{4}{x} - \dfrac{3}{x^2}$

$f(x) = 3x + 4x^{-1} - 3x^{-2}$

$f'(x) = 3 - 4x^{-2} + 6x^{-3}$

$\boxed{f'(x) = 3 - \dfrac{4}{x^2} + \dfrac{6}{x^3}}$

For each problem, find the indicated derivative with respect to x.

11) $f(x) = 2x^5 - 5x^4 - x^2 + 3x$ Find f''

$f'(x) = 10x^4 - 20x^3 - 2x + 3$

$\boxed{f''(x) = 40x^3 - 60x^2 - 2}$

12) $f(x) = \dfrac{3}{x^5} + 4e^x$ Find f''

$f(x) = 3x^{-5} + 4e^x$

$f'(x) = -15x^{-6} + 4e^x$

$f''(x) = 90x^{-7} + 4e^x$

$\boxed{f''(x) = \dfrac{90}{x^7} + 4e^x}$

Differentiate each function with respect to x.

13) $f(x) = -3x^4(-5x^5 + 2)$

$f(x) = 15x^9 - 6x^4$

$\boxed{f'(x) = 135x^8 - 24x^3}$

or

product rule

$-3x^4(-25x^4) + -12x^3(-5x^5+2)$

$75x^8 + 60x^8 - 24x^3$

$\boxed{f'(x) = 135x^8 - 24x^3}$

14) $f(x) = (2x^4 + 4)(-2x^4 + x^2 + 2)$

product rule

$(2x^4+4)(-8x^3+2x) + (8x^3)(-2x^4+x^2+2)$

$-16x^7 + 4x^5 - 32x^3 + 8x + -16x^7 + 8x^5 + 16x^3$

$\boxed{-32x^7 + 12x^5 - 16x^3 + 8x}$

15) $f(x) = \frac{4x^5}{3x^2+3}$ ← Quotient Rule → 16) $f(x) = \frac{5x^2+5}{2x^4-3}$

$$\frac{20x^4(3x^2+3) - 6x(4x^5)}{(3x^2+3)^2}$$

$$\frac{60x^6+60x^4-24x^6}{(3x^2+3)^2} = \boxed{\frac{36x^6+60x^4}{(3x^2+3)^2}}$$

$$\frac{10x(2x^4-3) - 8x^3(5x^2+5)}{(2x^4-3)^2}$$

$$\frac{20x^5-30x-40x^5-40x^3}{(2x^4-3)^2} = \boxed{\frac{-20x^5-40x^3-30x}{(2x^4-3)^2}}$$

17) $f(x) = (-4x^4+3)^4$ ← Chain rule → 18) $f(x) = (3x+2)^2$

$$4(-4x^4+3)^3 \cdot (-16x^3)$$

$$\boxed{-64x^3(-4x^4+3)^3}$$

$$2(3x+2)^1 \cdot (3)$$

$$\boxed{6(3x+2) \text{ or } 18x+12}$$

you could have multiplied out $(3x+2)^2$ and gotten the same answer

19) $f(x) = e^{2x^4}$ ← Chain rule → 20) $f(x) = \ln 2x^5$

$$e^{2x^4} \cdot 8x^3$$

$$\boxed{8x^3e^{2x^4}}$$

$$\frac{1}{2x^5} \cdot 10x^4$$

$$\frac{10x^4}{2x^5} = \boxed{\frac{5}{x}}$$

21) $f(x) = \sin 5x^2$ ← Chain rule → 22) $f(x) = 2\tan 5x$

$$(\cos 5x^2) \cdot 10x$$

$$\boxed{10x\cos 5x^2 \text{ or } 10x\cos(5x^2)}$$

$$2(\sec^2 5x) \cdot 5 \quad \boxed{\text{or}} \quad 2\frac{1}{\cos^2 5x} \cdot 5$$

$$\boxed{10\sec^2 5x \text{ or } 10\sec^2(5x)}$$

$$\boxed{\frac{10}{\cos^2 5x} \text{ or } \frac{10}{\cos^2(5x)}}$$

23) $f(x) = e^{3x}\cos 5x^3$ product + chain

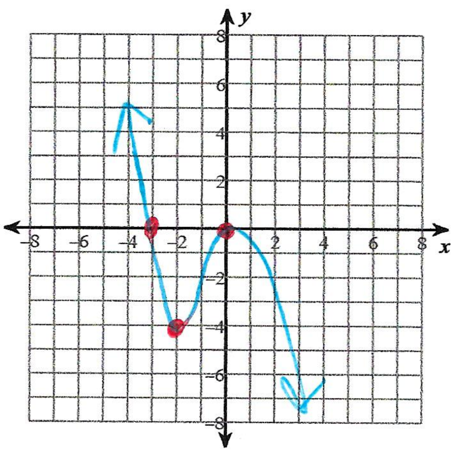
$$e^{3x} \cdot 3(\cos 5x^3) + e^{3x}(-\sin(5x^3) \cdot 15x^2)$$

$$3e^{3x}\cos(5x^3) - 15x^2e^{3x}\sin(5x^3)$$

$$\boxed{3e^{3x}(\cos 5x^3 - 5x^2\sin 5x^3)}$$

For each problem, find the: x and y intercepts, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

24) $y = -x^3 - 3x^2$



intercepts

$0 = -x^3 - 3x^2$

$0 = -x^2(x+3)$

$\boxed{x = 0 \text{ \& } x = -3}$

$y = -(0)^3 - 3(0)^2$

$\boxed{y = 0}$

Critical pts / inc + dec / min/max

$y' = -3x^2 - 6x$

$0 = -3x(x+2)$

$x = 0 \text{ \& } -2$

y': $-$ | $+$ | $-$ at -2, 0

$\boxed{\text{inc: } (-2, 0) \quad \text{dec } (-\infty, -2)(0, \infty)}$

max & min → plug into original

$\boxed{\text{max } (0,0) \quad \text{min } (-2, -4)}$

inflect / cc↑ cc↓

$y'' = -6x - 6$

$0 = -6x - 6$

$x = -1$

y'': $+$ | $-$ at -1

$\boxed{cc\uparrow \ (-\infty, -1) \quad cc\downarrow \ (-1, \infty)}$

inf pt → plug into original

$\boxed{\text{infpt } (-1, -2)}$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, and the intervals of time when the particle is moving left and moving right.

25) $s(t) = t^3 - 24t^2 + 144t$

$\boxed{v(t) = 3t^2 - 48t + 144}$

$\boxed{a(t) = 6t - 48}$

change direction?

$v(t) = 0$

$3t^2 - 48t + 144 = 0$

$3(t^2 - 16t + 48) = 0$

$3(t-4)(t-12) = 0$

$\boxed{t = 4 \text{ \& } 12 \text{ sec}}$

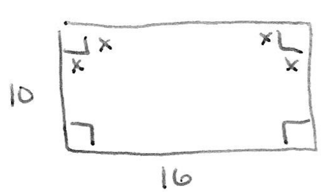
left / right?

$v(t)$: $+$ | $-$ | $+$ at 0, 4, 12

$\boxed{\text{left: } (4, 12) \quad \text{right: } (0, 4) \ (12, \infty)}$

Solve each optimization problem.

26) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?



height $= x$

length $= 16 - 2x$

width $= 10 - 2x$

Volume $= x(16-2x)(10-2x)$

$V = x(160 - 32x - 20x + 4x^2)$

$V = 4x^3 - 52x^2 + 160x$

$V' = 12x^2 - 104x + 160$

$0 = 12x^2 - 104x + 160$

$0 = 4(3x^2 - 26x + 40)$

$0 = 4(3x - 20)(x - 2)$

or use Quad for. App

$x = 20/3$ (too big)

$\boxed{x = 2 \text{ in}}$

27) A farmer wants to construct a rectangular pigpen using 400 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

Perimeter

$400 = 2x + y$

$400 - 2x = y$

Area $= xy$

$x \cdot (400 - 2x) = A$

$400x - 2x^2 = A$

$400 - 4x = A'$

$0 = 400 - 4x$

$\boxed{x = 100 \text{ ft} \quad \text{so } y = 200 \text{ ft}}$