15.1 Random Variables

A Random Variable is a quantity whose value depends on chance.
Random variables are represented by capital letters.

Here are some examples of random variables:

- X = the number of sixes obtained when a dice is rolled 3 times
- B = the number of babies is a pregnancy
- M = the mass of crisps in a packet
- T = the time taken for a runner to comlete 100m

There are two basic types of random variables:

1. Discrete Random Variables
   - These have a finite or countable number of possible values (e.g. X and B above)
   - Does not have to just positive integer values (e.g. shoe sizes of a set of students could have possible values of …4, 4.5, 5, 5.5, 6, 6.5, …)
2. Continuous Random Variables
   - These can take on any value in some interval (e.g. M and T above)

Consider the discrete random variable X, the number of sixes obtained when a dice is rolled 3 times.

- You can write to represent "the probability that the number of sixes is x" where x can take the values 0, 1, 2, and 3.

we can find $P(X=0), P(X=1)$
(capital)

Use capital letters for random variables.
Use lower case letters for the actual measured values.

Before calculators and computers random number tables were used to find random numbers.
See page 520.

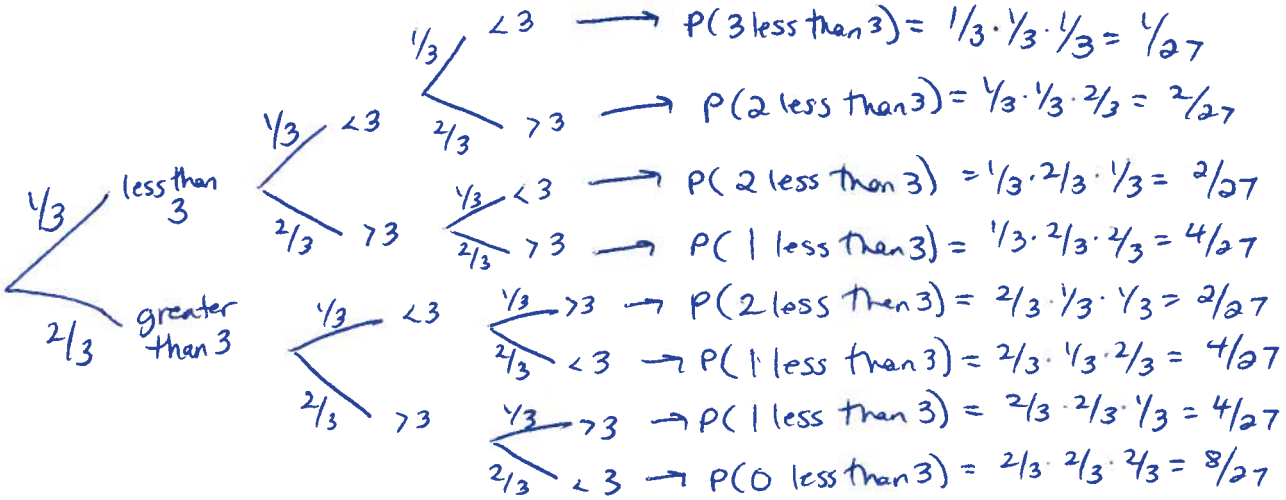
Probability distributions of discrete random variables

A **probability distribution** for a discrete random variable is a list of each possible value of the random variable and the probability that each outcome occurs.

Example: Let S be the random variable that represents the number of times a number less than 3 is rolled when a fair dice is rolled three times. Tabulate the probability distribution for S. ↑ $2/6 = 1/3$

Use a tree diagram to find the values of $P(S=0)$, $P(S=1)$, $P(S=2)$, and $P(S=3)$

s	0	1	2	3
$P(S=s)$	$8/27$	$12/27$	$6/27$	$1/27$



Notice that in the example the sum of the probabilities is… one

For any random variable X (Always one)

$$0 \le P(X=x) \le 1 \qquad \sum P(X=x) = 1$$

Sometimes $P(X=x)$ is replaced with just $P(x)$ or P_x - these mean the same thing.

What does $0 \le P(X=x) \le 1$ and $\sum P(X=x) = 1$ mean?

↳ means that a probability must always be between 0 + 1

$\sum P(X=x)$ means the sum of the probabilities will always be 1.

Example: The random variable X has the probability distribution

a. Find the value of q

b. Find $P(X \geq 3)$

x	1	2	3	4
$P(X=x)$	$5q$	$3q$	$6q$	q

plug in $q = 1/15$ → $1/3$ $\frac{1}{5}$ $2/5$ $1/15$

(a) $\sum P(X=x) = 1$

$5q+3q+6q+q = 1$

$15q = 1$

$q = 1/15$

(b) $P(X \geq 3)$

$= \frac{2}{5} + \frac{1}{15}$

$\frac{6}{15} + \frac{1}{15}$

$= \frac{7}{15}$

Exercise 15A

Expectation

The mean or expected value of a random variable X is the average value that we should expect for X over many trials of the experiment.

The mean or expected value of a random variable X is represented by $\mathrm{E}(X)$.

We would expect the mean to be the same in each case. Therefore we can find the mean or expected value of the random variable D by just multiplying each value of d by its respective probability (the equivalent of conducting the experiment just once).

The expected value of a random variable X is

$$\mathrm{E}(X) = \sum xP(X=x)$$

Using the probability distribution for the even S in the first example. What is the expected value of S? First let's do this by hand and then we can do it on the GDC.

$E(x) = \sum P(X=5) = 0(8/27) + 1(12/27) + 2(6/27) + 3(1/27)$

$= 0 + 12/27 + 12/27 + 3/27$

$= 1$ → meaning if 3 die are rolled a large # of times, the expected mean of rolls less than 3 would be 1.

Exercise 15B