Precalculus

Name Solutions

9.1-9.3 Conics Review

Use the information provided to write the standard form equation of each parabola.

1) Vertex at origin, Focus: $\left(0, \frac{3}{4}\right)$

$x^2 = 4py \quad p = \frac{3}{4}(4p)$

$x^2 = 4(3/4)y$

$x^2 = 3y$

* Remember p = distance from vertex to focus!

2) Vertex: $(2, 0)$, Focus: $(0, 0)$

h, k

$(y-k)^2 = 4p(x-h) \quad p = -2$ (left)

$(y-0)^2 = 4(-2)(x-2)$

$y^2 = -8(x-2)$

3) Vertex: $(1, 2)$, Directrix: $x = -3$

also distance from vertex to directrix

h, k

$(y-k)^2 = 4p(x-h) \quad p = 4$ (right)

$(y-2)^2 = 4(4)(x-1)$

$(y-2)^2 = 16(x-1)$

4) Opens up or down, Vertex: $(6, -10)$, Passes through: $(4, -9)$

h k

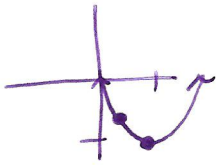
x y

$(x-h)^2 = 4p(y-k)$

$(4-6)^2 = 4p(-9+10)$

$4 = 4p$

$p = 1$



$(x-6)^2 = 4(1)(y+10)$

$(x-6)^2 = 4(y+10)$

5) Find the vertex, foci, and directrix of $-8(x+2) = (y+3)^2$. Then sketch a graph.

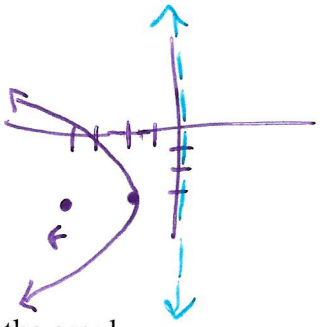
$-8(x+2) = (y+3)^2$

$4p(x-h) = (y-k)^2$

$4p = -8$

$p = -2$

Vertex $(-2, -3)$



* count "p" to find ...

Focus $(-4, -3)$

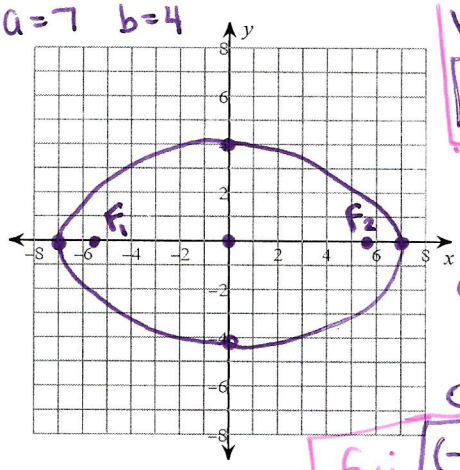
Directrix $x = 0$

Identify the vertices and foci of each. Then sketch the graph.

6) $\frac{x^2}{49} + \frac{y^2}{16} = 1$

Center (0,0)

$a = 7 \quad b = 4$



Vertices: $(-7, 0)(7, 0)$

foci:

$c^2 = a^2 - b^2$

$c^2 = 7^2 - 4^2$

$c^2 = 33$

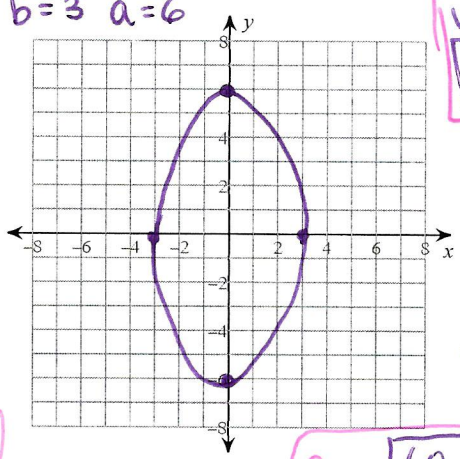
$c = \pm\sqrt{33} \approx 5.7$

foci $(-\sqrt{33}, 0), (\sqrt{33}, 0)$

7) $\frac{x^2}{9} + \frac{y^2}{36} = 1$

Center (0,0)

$b = 3 \quad a = 6$



Vertices: $(0, 6)(0, -6)$

foci:

$c^2 = a^2 - b^2$

$c^2 = 6^2 - 3^2$

$c^2 = 27$

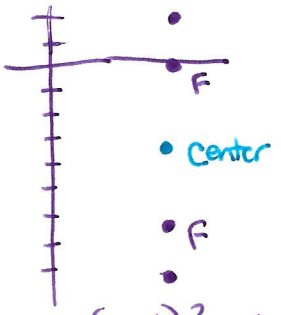
$c = \pm\sqrt{27} = \pm\sqrt{9}\sqrt{3}$

$c = \pm 3\sqrt{3} \approx 5.2$

foci $(0, 3\sqrt{3}), (0, -3\sqrt{3})$

Use the information provided to write the standard form equation of each ellipse.

8) Vertices: (8, 2), (8, –8)
Foci: (8, 0), (8, –6)



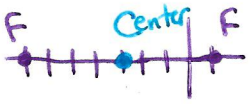
Vertical (see sketch)
Center (midpoint) (8, –3)
$a = 5$ (distance to vertex)
$c = 3$ (distance to foci)
$c^2 = a^2 - b^2$
$3^2 = 5^2 - b^2$ $b = 4$

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

$\frac{(x-8)^2}{16} + \frac{(y+3)^2}{25} = 1$

9) Foci: (1, 0), (–7, 0)
Major Axis Length: 10 units
Minor Axis Length: 6 units

$2a = 10$ $a = 5$
$2b = 6$ $b = 3$



Horizontal (see sketch)
Center (midpoint) (–3, 0)

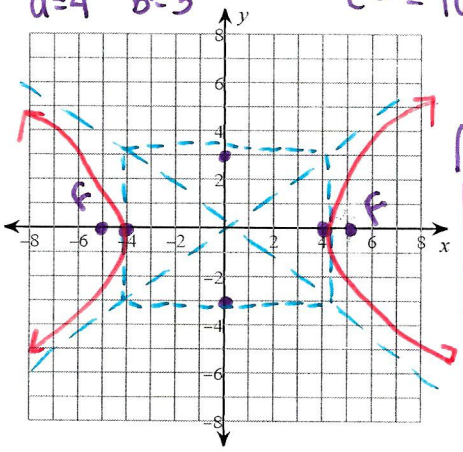
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x+3)^2}{25} + \frac{y^2}{9} = 1$

Identify the vertices and foci of each. Then sketch the graph.

10) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

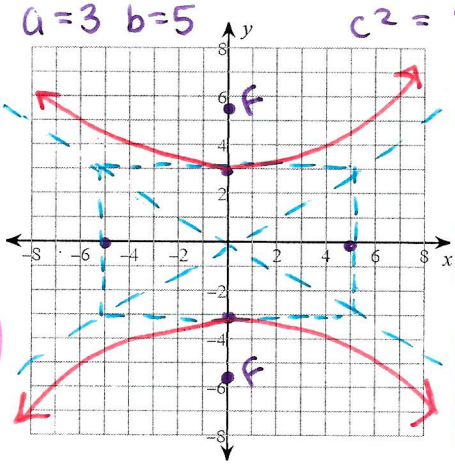
Horizontal (x^2 first)
$a = 4$ $b = 3$
$c^2 = a^2 + b^2$
$c^2 = 16 + 9$
$c^2 = 25$
$c = 5$



Center (0,0)
Vertices (4,0) (–4,0)
Foci (5,0) (–5,0)

11) $\frac{y^2}{9} - \frac{x^2}{25} = 1$

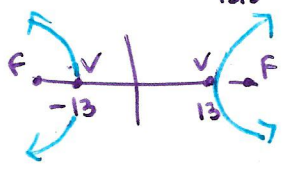
Vertical (y^2 first)
$a = 3$ $b = 5$
$c^2 = a^2 + b^2$
$c^2 = 9 + 25$
$c^2 = 34$
$c = \sqrt{34} \approx 5.8$



Center (0,0)
Vertices (0,3) (0,–3)
Foci $(0, \sqrt{34})$ $(0, -\sqrt{34})$

Use the information provided to write the standard form equation of each hyperbola.

12) Vertices: (13, 0), (–13, 0)
Foci: $(\sqrt{233}, 0)$, $(-\sqrt{233}, 0)$
15.3 −15.3



Horizontal (see sketch)
Center (0,0)
$a = 13$ (distance to vertex)
$c = \sqrt{233}$ (distance to foci)

$c^2 = a^2 + b^2$
$(\sqrt{233})^2 = 13^2 + b^2$
$233 = 169 + b^2$
$b = 8$

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\frac{x^2}{169} - \frac{y^2}{64} = 1$

13) Transverse Axis Length: 8 units
Foci: (2, –3), (2, –13)

$2a = 8$
$a = 4$

Vertical (see sketch)
Center (2, –8)
$c = 5$ (distance to foci)

$c^2 = a^2 + b^2$
$25 = 16 + b^2$
$b = 3$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\frac{(y+8)^2}{16} - \frac{(x-2)^2}{9} = 1$

Without putting them into Standard Form, classify each of the following equations as a parabola, circle, ellipse, or hyperbola. Explain why.

14) $5x^2 - 2y^2 + 10x - 4y + 17 = 0$

squared terms opposite signs

hyperbola

15) $-4y^2 + 5x + 3y + 7 = 0$

only 1 squared term

parabola

16) $3x^2 + 2y^2 - 12x + 12y + 29 = 0$

squared terms same signs

ellipse

17) $4x^2 + 4y^2 - 4x + 8y = 11$

squared terms same sign, same #

circle

18) $2x^2 - 2y^2 = 2$

Squared terms opposite signs

hyperbola

19) $3(x + 2)^2 = y$

x^2 → only 1 squared term

parabola

Answer the following application questions.

20) The revenue R in dollars generated by the sale of x units of a patio furniture set is given by $(x - 106)^2 = -\frac{4}{5}(R - 14045)$. Approximate the number of sales that will maximize the revenue.

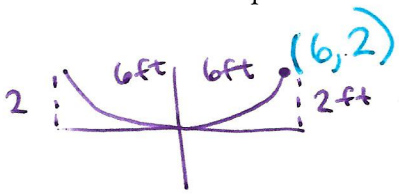
maximum = vertex

(106, 14045)

Sales $\quad$ $\$$

106 units

21) A satellite dish is in the shape of of a parabolic surface. The satellite dish has a diameter of 12 feet and a depth of 2 feet. How far from the base of the dish should the receiver be placed?



Find p

$x^2 = 4py$

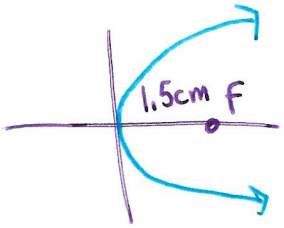
$6^2 = 4p(2)$

$36 = 8p$

$p = 4.5$

4.5 feet

22) The light bulb in a flashlight is at the focus of its parabolic reflector, 1.5 centimeters from the vertex of the reflector. Assuming the reflector is pointed towards the right, write an equation of a cross section of the flashlight's reflector with its focus on the positive x axis and its vertex at the origin .



$p = 1.5$ (distance to focus - positive since opens right)

$y^2 = 4px$

$y^2 = 4(1.5)x$

$y^2 = 6x$