Ch. 9 Non-Calc Review

1) ⓐ $\int (4x^3 - 8x + 6)\,dx = \frac{1}{4}(4x^4) - \frac{1}{2}(8x^2) + 6x + C$

$\boxed{x^4 - 4x^2 + 6x + C}$

ⓑ $\int \sqrt[3]{x^4}\,dx = \int x^{4/3}\,dx = \boxed{3/7\, x^{7/3} + C}$

ⓒ $\int 3/x^4\,dx = \int 3x^{-4}\,dx = -\frac{1}{3}(3x^{-3}) + C = -x^{-3} + C$

$= \boxed{\frac{-1}{x^3} + C}$

ⓓ $\int \frac{5x^4 - 3x}{6x^2}\,dx = \int \left(\frac{5x^4}{6x^2} - \frac{3x}{6x^2}\right) dx$

$\int \left(\frac{5}{6}x^2 - \frac{1}{2}\cdot\frac{1}{x}\right) dx = \frac{1}{3}\cdot\frac{5}{6}x^3 - \frac{1}{2}\ln|x| + C$

$\boxed{\frac{5}{18}x^3 - \frac{1}{2}\ln|x| + C}$

ⓔ $\int e^{4x}\,dx = \int e^u \cdot \frac{1}{4}\,du$

$u = 4x$

$du = 4dx$ $\quad = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C = \boxed{\frac{1}{4}e^{4x} + C}$

$\frac{1}{4}du = dx$

ⓕ $\int x^2(x^3+1)^4\,dx = \int u^4 \cdot \frac{1}{3}\,du$

$u = x^3 + 1$

$du = 3x^2\,dx$ $\quad = \frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{1}{5}u^5 + C = \boxed{\frac{1}{15}(x^3+1)^5 + C}$

$\frac{1}{3}du = x^2\,dx$

ⓖ $\int \frac{1}{2x+3}\,dx = \int \frac{1}{u}\cdot\frac{1}{2}\,du$

$u = 2x+3$

$du = 2dx$ $\quad = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u| + C = \boxed{\frac{1}{2}\ln|2x+3| + C}$

$\frac{1}{2}du = dx$

ⓗ $\int \frac{\ln x}{x}\,dx = \int \ln x \cdot \frac{1}{x}\,dx = \int u \cdot du = \frac{1}{2}u^2 + C$

$u = \ln x$

$du = \frac{1}{x}\,dx$ $\quad \boxed{\frac{1}{2}(\ln x)^2 + C}$

(i) $\int (3x^2+1)(6x)\,dx = \int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}(3x^2+1)^2 + C}$

$u = 3x^2+1$

$du = 6x\,dx$

(j) $\int \frac{2e^x}{e^x+3}\,dx = \int 2e^x \cdot \frac{1}{e^x+3}\,dx$

$u = e^x + 3$

$du = e^x\,dx$

$2du = 2e^x\,dx$

$= \int \frac{1}{u}\cdot 2\,du = 2\int \frac{1}{u}\,du = 2\ln|u| + C$

$\boxed{2\ln|e^x+3| + C}$

(k) $\int 3\sqrt{2x-5}\,dx = 3\int (2x-5)^{1/2}\,dx$

$u = 2x-5$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= 3\int u^{1/2}\cdot \frac{1}{2}du = 3/2\int u^{1/2}\,du = 3/2 \cdot \frac{2}{3}u^{3/2} + C$

$\boxed{(2x-5)^{3/2} + C}$

(l) $\int 2xe^{2x^2}\,dx = \int e^u \cdot \frac{1}{2}du$

$u = 2x^2$

$du = 4x\,dx$

$\frac{1}{2}du = 2x\,dx$

$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{2x^2} + C}$

(2) (a) $\int_0^2 (3x^2-6)\,dx = 3\cdot\frac{1}{3}x^3 - 6x\Big]_0^2 = x^3 - 6x\Big]_0^2$

$[(2)^3 - 6(2)] - [0^3 - 6(0)] = 8 - 12 = \boxed{-4}$

(b) $\int_4^{16} \frac{4}{\sqrt{t}}\,dt = \int_4^{16} 4\cdot t^{-1/2}\,dt = 4\cdot 2t^{1/2}\Big]_4^{16} = 8\sqrt{t}\Big]_4^{16}$

$8\cdot\sqrt{16} - 8\sqrt{4} = 8\cdot 4 - 8\cdot 2 = 32 - 16 = \boxed{16}$

(c) $\int_1^{e^2} \frac{4}{x}\,dx = \int_1^{e^2} 4\cdot\frac{1}{x}\,dx = 4\cdot\ln|x|\Big]_1^{e^2}$

$4\ln e^2 - 4\ln 1 = 4\cdot 2 - 4\cdot 0 = \boxed{8}$

(d) $\int_0^1 6x\,e^{3x^2+3}\,dx$

$u = 3x^2+3$

$du = 6x\,dx$

if $x=1$ $u=6$

if $x=0$ $u=3$

$$\Rightarrow \int_3^6 e^u\,du = e^u\Big]_3^6 = \boxed{e^6 - e^3 \text{ or } e^3(e^3-1)}$$

(e) $\int_{-1}^1 (3x-1)^3\,dx$

$u = 3x-1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

if $x=1$ $u=2$

if $x=-1$ $u=-4$

$$\Rightarrow \int_{-4}^2 u^3 \cdot \tfrac{1}{3}du$$

$$= \frac{1}{3}\int_{-4}^2 u^3\,du = \frac{1}{3}\cdot\frac{1}{4}u^4\Big]_{-4}^2 = \frac{1}{12}\left[2^4-(-4)^4\right]$$

$$\frac{1}{12}\left[16-256\right] = \frac{1}{12}(-240) = \boxed{-20}$$

(f) $\int_0^2 \frac{1}{2x+1}\,dx$

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

if $x=2$ $u=5$

$x=0$ $u=1$

$$\Rightarrow \int_1^5 \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\int_1^5 \frac{1}{u}du = \frac{1}{2}\ln u\Big]_1^5$$

$$\frac{1}{2}\left[\ln 5 - \ln 1\right]$$

$$\frac{1}{2}\left[\ln 5 - 0\right] =$$

$$\frac{1}{2}\ln 5$$

$$\boxed{\frac{\ln 5}{2}}$$

(3) (a) $\int_1^2 (x^2-1)\,dx$

(b) $\frac{1}{3}x^3 - x \Big]_1^2 = \left(\frac{1}{3}(2)^3 - 2\right) - \left(\frac{1}{3}(1)^3 - 1\right)$

$= \frac{8}{3} - 2 - \frac{1}{3} + 1 = \frac{7}{3} - 1 = \frac{7}{3} - \frac{3}{3} = \boxed{\frac{4}{3}}$

(c) $\int_1^2 (x^2-1)\,dx + \left|\int_{-1}^1 (x^2-1)\,dx\right|$

or $\int_1^2 (x^2-1)\,dx - \int_{-1}^1 (x^2-1)\,dx$

* since I know $\int_{-1}^1$ is negative (since it's under the axis) if I subtract it from the 1st integral it will be a positive value (minus a negative = positive)

(d) $\boxed{\int_1^2 \pi (x^2-1)^2\,dx}$

(4) $f'(x) = 3x - 2$

$f(x) = 3 \cdot \frac{1}{2}x^2 - 2x + C$

$f(x) = 3/2\, x^2 - 2x + C$ if (2,6) is on the curve...

$6 = 3/2(2)^2 - 2(2) + C$

$6 = 6 - 4 + C$

$C = 4$ $\Rightarrow$ $\boxed{f(x) = 3/2\, x^2 - 2x + 4}$

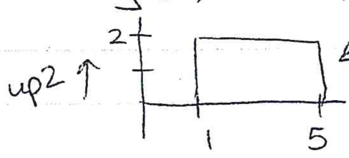
(5) $\int_1^5 f(x)\,dx = 20$

(a) $\int_1^5 \frac{1}{4} f(x)\,dx = \frac{1}{4}\int_1^5 f(x)\,dx = \frac{1}{4}(20) = \boxed{5}$

(b) $\int_1^5 [f(x) + 2]\,dx$

this adds a 4x2 area to my integral

$20 + 8 = \boxed{28}$



(6) $\int 4e^{2t}+2 \quad = \int 4e^{2t} + \int 2$

$u = 2t \qquad = \int e^{u}\cdot 2du + \int 2$

$du = 2dt$

$2du = 4dt \qquad = 2\int e^{u}du + \int 2 \quad = \quad 2e^{u} + 2t + C$

$2e^{2t} + 2t + C = S(t)$

when $t=0$ $\qquad 8 = 2e^{2(0)} + 2(0) + C$

$S = 8 \qquad 8 = 2(1) + 0 + C$

$C = 6 \qquad \boxed{2e^{2t} + 2t + 6 = S(t)}$

(7) $\int_{1}^{K} \frac{1}{2x-1}\,dx = \ln 5$

$u = 2x-1 \qquad \int_{1}^{2k-1} \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\int_{1}^{2k-1}\frac{1}{u}du$

$du = 2dx$

$\frac{1}{2}du = dx \qquad \frac{1}{2}\ln|u|\Big]_{1}^{2k-1}$

if $x=k$ $\quad u = 2k-1$

if $x=1$ $\quad u=1$

$\frac{1}{2}\Big[\ln(2k-1) - \underbrace{\ln 1}_{\text{equals } 0}\Big] = \ln 5$

$\frac{1}{2}\ln(2k-1) = \ln 5$

$\ln(2k-1)^{1/2} = \ln 5$

$(2k-1)^{1/2} = 5$

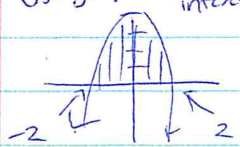
$2k-1 = 25$

$2k = 26$

$\boxed{k = 13}$

Ch. 9 Calc Review

1) use graph to find intercepts



or

$4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$$\int_{-2}^{2} \pi (4-x^2)^2\,dx = \int_{-2}^{2} \pi (16 - 8x^2 + x^4)\,dx$$

$$\pi \left[16x - \tfrac{8}{3}x^3 + \tfrac{1}{5}x^5\right]_{-2}^{2} = \pi\left[17\tfrac{1}{15} - (-17\tfrac{1}{15})\right] = 34\tfrac{2}{15}\pi \text{ Exact}$$

Approx: $34.1\overline{3}\pi = 107.2$

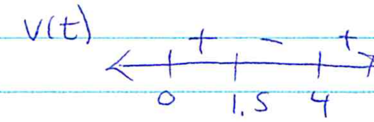
2) ⓐ $v(t) = 2t^2 - 11t + 12$

$a(t) = v'(t) = 4t - 11$

ⓑ $2t^2 - 11t + 12 = 0$

$(2t-3)(t-4) = 0$

$t = 1.5 \qquad t = 4$ or Quad form.



left $(1.5, 4)$ so $\boxed{a = 1.5 \quad b = 4}$

ⓒ option 1: $\int_2^5 |2t^2 - 11t + 12|\,dt = 7.83$

option 2:

object moves left from 2 to 4 and right from 4 to 5

$$\left|\int_2^4 (2t^2 - 11t + 12)\,dt\right| + \int_4^5 (2t^2 - 11t + 12)\,dt$$

$$\left|\tfrac{2}{3}t^3 - \tfrac{11}{2}t^2 + 12t\right]_2^4\Big| + \tfrac{2}{3}t^3 - \tfrac{11}{2}t^2 + 12t\Big]_4^5$$

$$\left|-4\tfrac{2}{3}\right| + 3\tfrac{1}{6} = 7\tfrac{5}{6} = 7.83$$

3) (a) tangent line to $f(x) = x^3 - 2$ at $x = -1$

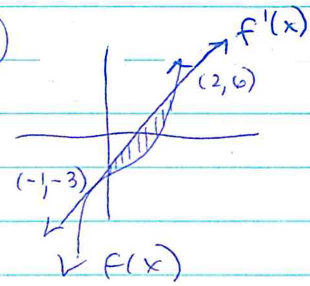
$f(-1) = (-1)^3 - 2 = -3$ $\quad (-1, -3)$

$m = f'(-1) = 3x^2 = 3(-1)^2 = 3$

$\boxed{y + 3 = 3(x+1)}$ or $y = 3x$

(b) Graph to find intersection $(2, 6)$

(c)



(d) $\int_{-1}^{2} \left(3x - (x^3 - 2)\right) dx$

$\boxed{6.75}$