REVIEW SET 5A

1 Identify the following sequences as arithmetic, geometric, or neither:

a $7, -1, -9, -17, \ldots$ **b** $9, 9, 9, 9, \ldots$ **c** $4, -2, 1, -\frac{1}{2}, \ldots$

d $1, 1, 2, 3, 5, 8, \ldots$ **e** the set of all multiples of 4 in ascending order.

2 Find k if $3k$, $k-2$, and $k+7$ are consecutive terms of an arithmetic sequence.

3 Show that $28, 23, 18, 13, \ldots$ is an arithmetic sequence. Hence find u_n and the sum S_n of the first n terms in simplest form.

4 Find k given that 4, k, and $k^2 - 12$ are consecutive terms of a geometric sequence.

5 Determine the general term of a geometric sequence given that its sixth term is $\frac{16}{3}$ and its tenth term is $\frac{256}{3}$.

6 Insert six numbers between 23 and 9 so that all eight numbers are in arithmetic sequence.

7 Find the 8th term of each of the following sequences:

a $5, 1, \frac{1}{5}, \ldots$ **b** $-11, -8\frac{1}{2}, -6, \ldots$ **c** $a, a-d, a-2d, \ldots$

8 At the start of the dry season, Yafiah's 3000 L water tank is full. She uses 183 L of water each week to water her garden.

a Find the amount of water left in the tank after 1, 2, 3, and 4 weeks.

b Explain why the amount of water left in the tank after n weeks forms an arithmetic sequence.

c When does Yafiah's tank run out of water?

9 Find the sum of:

a $14 + 11 + 8 + \ldots + (-55)$ **b** $3 + 15 + 75 + \ldots$ to 10 terms

10 Consider the arithmetic sequence $12, 19, 26, 33, \ldots$

a Find the 8th term of the sequence.

b Find the sum of the first 10 terms of the sequence.

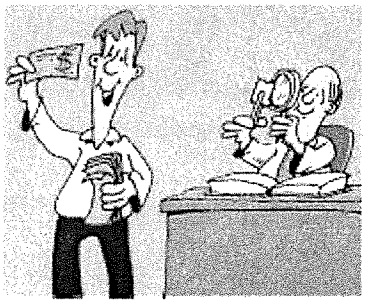
c The sum of the first n terms is 915. Find the value of n.

11 Val receives a \$285 000 superannuation payment when she retires. She finds the following investment rates are offered:

Bank A : 6% p.a. compounded quarterly *Bank B* : $5\frac{3}{4}\%$ p.a. compounded monthly.

Compare the interest that would be received from these banks over a ten year period. In which bank should Val deposit her superannuation?

12 Sven sells his stamp collection and deposits the proceeds of \$8700 in a term deposit account for nine months. The account pays $9\frac{3}{4}\%$ p.a. compounded monthly. How much interest will he earn over this period?



13 **a** Find the future value of a truck which is purchased for \$135 000 and depreciates at 15% p.a. for 5 years.

b By how much does it depreciate?

14 Ena currently has £7800, and wants to buy a car valued at £9000. She puts her money in an account paying 4.8% p.a. compounded quarterly. When will she be able to buy the car?

REVIEW SET 5B

1 A sequence is defined by $u_n = 6(\frac{1}{2})^{n-1}$.

a Prove that the sequence is geometric. **b** Find u_1 and r.

c Find the 16th term to 3 significant figures.

2 Consider the sequence $24, 23\frac{1}{4}, 22\frac{1}{2}, ..., -36$. Find:

a the number of terms in the sequence. **b** the value of u_{35} for the sequence.

c the sum of the terms in the sequence.

3 Find the sum of:

a $3 + 9 + 15 + 21 + ...$ to 23 terms **b** $24 + 12 + 6 + 3 + ...$ to 12 terms.

4 List the first five terms of the sequence:

a $\{(\frac{1}{3})^n\}$ **b** $\{12 + 5n\}$ **c** $\left\{\frac{4}{n+2}\right\}$

5 **a** What will an investment of €6000 at 7% p.a. compound interest amount to after 5 years?

b What part of this is interest?

6 Find the first term of the sequence $24, 8, \frac{8}{3}, \frac{8}{9}, ...$ which is less than 0.001.

7 A geometric sequence has $u_6 = 24$ and $u_{11} = 768$.

a Determine the general term of the sequence. **b** Hence find u_{17}.

c Find the sum of the first 15 terms.

8 The nth term of a sequence is given by the formula $u_n = 4n - 7$.

a Find the value of u_{10}.

b Write down an expression for $u_{n+1} - u_n$ and simplify it.

c Hence explain why the sequence is arithmetic.

d Evaluate $u_{15} + u_{16} + u_{17} + ... + u_{30}$.

9 **a** Determine the number of terms in the sequence $128, 64, 32, 16, ..., \frac{1}{512}$.

b Find the sum of these terms.

10 For the geometric sequence $180, 60, 20, ...$, find:

a the common ratio for this sequence. **b** the 6th term of the sequence.

c the least number of terms required for the sum of the terms to exceed 269.9.

11 Before leaving overseas on a three year trip to India, I leave a sum of money in an account that pays 6% p.a. compounded half-yearly. When I return from the trip, there is €5970.26 in my account. How much interest has been added since I have been away?

12 Megan deposits £3700 in an account paying interest compounded monthly for two years. If she ends up with £4072, what rate of interest did Megan receive?

13 Kania purchases office equipment valued at $17 500.

a At the end of the first year, the value of the equipment is $15 312.50. Find the rate of depreciation.

b If the value of the equipment continued to depreciate at the same rate, what would it be worth after $3\frac{1}{2}$ years?

REVIEW SET 5C

1 A sequence is defined by $u_n = 68 - 5n$.

a Prove that the sequence is arithmetic.

b Find u_1 and d.

c Find the 37th term of the sequence.

d State the first term of the sequence which is less than -200.

2 **a** Show that the sequence $3, 12, 48, 192, \ldots$ is geometric.

b Find u_n and hence find u_9.

3 Find the general term of the arithmetic sequence with $u_7 = 31$ and $u_{15} = -17$. Hence, find the value of u_{34}.

4 Consider the sequence $24, a, 6, \ldots$

Find the value of a if the sequence is: **a** arithmetic **b** geometric.

5 Find the 10th term of the sequence:

a $32, 25, 18, 11, \ldots$ **b** $\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots$

6 There were originally 3000 koalas on Koala Island. Since then, the population of koalas on the island has increased by 5% each year.

a How many koalas were on the island after 3 years?

b How long will it take for the population to exceed 5000?

7 Find the formula for u_n, the general term of:

a $86, 83, 80, 77, \ldots$ **b** $\frac{3}{4}, 1, \frac{7}{6}, \frac{9}{7}, \ldots$ **c** $100, 90, 81, 72.9, \ldots$

Hint: One of these sequences is neither arithmetic nor geometric.

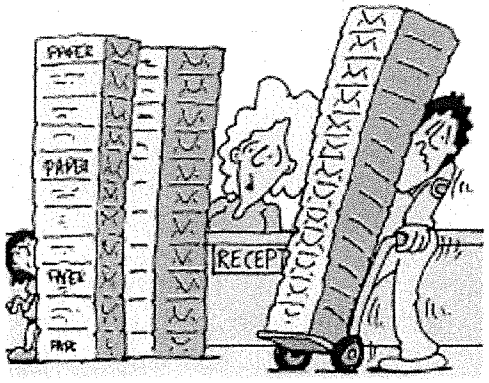
8 Find the first term of the sequence $5, 10, 20, 40, \ldots$ which exceeds 10 000.

9 $-1, k, k^2 - 7$ are consecutive terms of an arithmetic sequence. Find k.

10 Each year, a school manages to use only 90% as much paper as the previous year. In the year 2000, they used 700 000 sheets of paper.

a Find how much paper the school used in the years 2001 and 2002.

b How much paper did the school use in total in the decade from 2000 to 2009?



11 Find the final value of a compound interest investment of €8000 after 7 years at 3% p.a. with interest compounded annually.

12 Ned would like to have £15 000 in 3 years' time to install a swimming pool. His bank pays 4.5% p.a. interest, compounded half-yearly. How much does Ned need to deposit now?

13 A motorbike, purchased for £2300, was sold for £1300 after 4 years. Calculate the average annual rate of depreciation.

Review exercise

Paper 1 style questions

EXAM-STYLE QUESTIONS

1 **a** At what interest rate, compounded annually, would you need to invest $500 in order to have $625 in 2 years?

b At this interest rate, how long would it take for your $500 to double in value?

2 **a** In a city, house prices have increased by 2.3% each year for the last three years. If a house cost USD 240 000 three years ago, calculate its value today, to the nearest dollar.

b In another city, a house worth USD 200 000 three years ago is now valued at USD 214 245. Calculate the yearly percentage increase in the value of this house.

3 Joseph decides to invest GBP 1200 of his money in a savings account which pays interest at 4.3%, compounded annually.

a How much interest will the GBP 1200 earn after 4 years?

b For how many years must Joseph invest his GBP 1200 in order to earn at least GBP 250 in interest?

c How long will it take for his money to double?

4 The exchange rate from US dollars (USD) to euros (EUR) is given by 1 USD = 0.753 EUR. Give the answers to the following correct to **two** decimal places.

a Convert 125 US dollars to euros.

b Roger receives 800 Australian dollars (AUD) for 610 EUR. Calculate the value of the US dollar in Australian dollars.

5 In 2010, Heidi joined a golf club. The fees were £1500 a year. Each year the fees increase by 3.5%.

a Calculate, **to the nearest £1**, the fees in 2012.

b Calculate the **total** fees for Heidi, who joined the golf club in 2010 and remained a member for five years.

6 Emma places €18 000 in a bank account that pays a nominal interest rate of 4.5% per annum, compounded quarterly.

a Calculate the amount of money that Emma would have in her account after 15 years. Give your answer correct to the nearest euro.

b After a period of time she decides to withdraw the money from this bank. There is €19 862.21 in her account. Find the number of months that Emma had left her money in the account.

Paper 2 style questions

EXAM-STYLE QUESTIONS

1 A lottery is offering prizes in a new competition. The winner may choose one of three options.

Option one: \$2000 each week for 10 weeks.

Option two: \$1000 in the first week, \$1250 in the second week, \$1500 in the third week, increasing by \$250 each week for a total of 10 weeks.

Option three: \$15 in the first week, \$30 in the second week, \$60 in the third week continuing to double for a total of 10 weeks.

a Calculate the amount you receive in the eighth week, if you select
  i **option two**
  ii **option three**.

b What is the total amount you receive if you select **option two**?

c Which option has the greatest total value?

2 On Betty's 16th birthday she was given an allowance from her parents. She was given four choices.

Choice A: \$150 every month of the year.

Choice B: A fixed amount of \$1600 at the beginning of the year, to be invested at an interest rate of 10% per annum, compounded monthly.

Choice C: \$105 the first month and an increase of \$10 every month thereafter.

Choice D: \$120 the first month and an increase of 5% every month.

a Assuming that Betty does not spend any of her allowance during the year, calculate, for each of the choices, how much money she would have at the end of the year.

b Which of the choices do you think Betty should choose? Give a reason for your answer.

c On her 17th birthday Betty invests \$1500 in a bank that pays interest at r% per annum compounded annually. She would like to buy a car costing \$1800 on her 20th birthday. What rate will the bank have to offer her to enable her to buy the car?

EXAM-STYLE QUESTIONS

Give all answers in this question correct to the **nearest** dollar.

3 Cynthia wants to buy a house. She can choose between two different payment options. Both options require her to pay for the house in **20** yearly installments.

Option 1: The first installment is \$2000. Each installment is \$250 more than the one before.

Option 2: The first installment is \$2800. Each installment is 5% more than the one before.

a If Cynthia chooses **option 1,**

i write down the values of the second and third installments

ii calculate the value of the final installment

iii show that the **total amount** that Cynthia would pay for the house is \$87 500.

b If Cynthia chooses **option 2,**

i find the value of the second installment

ii show that the value of the fifth installment is \$3403.42.

c Cynthia knows that the **total amount** she would pay for the house is not the same for both options. She wants to spend the least amount of money. Find how much she will save by choosing the cheaper option.

4 The first three terms of an arithmetic sequence are $3k + 1$, $5k$ and $6k + 4$

a Show that $k = 5$.

b Find the values of the first three terms of the sequence.

c Write down the value of the common difference.

d Calculate the 15th term of the sequence.

e Find the sum of the first 20 terms of the sequence.

5 Arthur is starting his first job. He will earn a salary of 28 000 GBP in the first year and his salary will increase by 4% every year.

a Calculate how much Arthur will earn in his 4th year of work.

Arthur spends 24 000 GBP of his earnings in his first year of work. For the next few years, inflation will cause Arthur's living expenses to rise by 5% per year.

b i Calculate the number of years it will be before Arthur is spending more than he earns.

ii By how much will Arthur's spending be greater than his earnings in that year?

6 A geometric progression G_1 has 2 as its first term and 4 as its common ratio.

a The sum of the first n terms of G_1 is 11 184 810. Find n.

A second geometric progression G_2 has the form $2, \frac{2}{5}, \frac{2}{25}, \frac{2}{125}, \cdots$

b State the common ratio for G_2.

c Calculate the sum of the first 10 terms of G_2.