IB Math SL Name Solutions

Additional Chapter 7 Review

Write equations to optimize the given situations. Then solve.

1) Find two nonnegative numbers whose sum is 9 and so that the product of one number and the square of the other number is a maximum.

X = 1st number

y = 2nd number

$$\boxed{\begin{array}{c} x+y=9 \\ xy^2 = \text{max} \end{array}}$$

$x = 9-y$

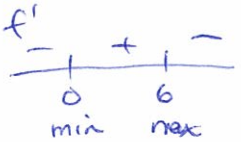
$(9-y)y^2 = \text{max}$

$9y^2 - y^3 = \text{max}$

$18y - 3y^2 = 0$

$3y(6-y) = 0$

$y=0,\ y=6$



$\boxed{y=6, x=3}$

2) An open rectangular box with square base is to be made from 48 cm^2 of material. What dimensions will result in a box with the largest possible volume ?

$SA = x^2 + 4xh$

$\boxed{48 = x^2 + 4xh}$

$\frac{48-x^2}{4x} = h$

$\boxed{x^2 h = V}$

$x^2\left(\frac{48-x^2}{4x}\right) = V$

$\frac{48x - x^3}{4} = V$

$12x - \frac{1}{4}x^3 = V$

$12 - 3/4x^2 = 0$

$12 = 3/4x^2$

$x = \pm 4$

$\boxed{4 \times 4 \times 2 \text{ cm}}$

3) A normal 12 oz can of soda has a volume of 355 cm^2 Find the dimensions (radius r and height h) that minimize the material used to construct the can. Round dimensions to the nearest tenth.

V = (Area of Base) h

$V = \pi r^2 h$

$\boxed{355 = \pi r^2 h}$

$\frac{355}{\pi r^2} = h$

SA = 2 (Area of Base) + Area of middle

$SA = 2(\pi r^2) + 2\pi r \cdot h$

$\boxed{SA = 2\pi r^2 + 2\pi r h}$

$SA = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right)$

$SA = 2\pi r^2 + 710r^{-1}$

$6 = 4\pi r - 710r^{-2}$

$0 = r(4\pi - 710r^{-3})$

$r \neq 0 \quad \frac{710}{r^3} = 4\pi$

$r^3 = \frac{710}{4\pi}$

$$\boxed{\begin{array}{l} r \doteq 3.8 \text{ cm} \\ h \doteq 7.7 \text{ cm} \end{array}}$$

For each problem, find all points of absolute minima and maxima on the given interval.

4) $y = x^2 - 4x + 6;\quad [0, 3]$

$y' = 2x-4$

$0 = 2x-4$

$x = 2$

$f(0) = 6$

$f(2) = 2$

$f(3) = 3$

Abs max (0,6)

Abs min (2,2)

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$, the acceleration function $a(t)$, the times t when the particle changes directions, the intervals of time when the particle is moving left and moving right, and the times t when the acceleration is 0. When is velocity increasing?

5) $s(t) = t^3 - 12t^2$

$v(t) = 3t^2 - 24t$

$0 = 3t^2 - 24t$

$0 = 3t(t-8)$

~~$t=0$~~, $t = 8$ sec

at rest

(change direction)

v(t): 0 | − | 8 | +

~~s(8) = -256~~

min (8, −256)

left [0,8) right (8,∞)

$a(t) = 6t - 24$

$0 = 6t - 24$

$24 = 6t$

$t = 4$ sec

a(t): − | 4 | +

inc. (4,∞)

6) $s(t) = t^3 - 24t^2 + 144t$

$v(t) = 3t^2 - 48t + 144$

$0 = 3(t - 16t + 48)$

$0 = 3(t-4)(t-12)$

$t = 4, t = 12$ sec

at rest

(change in direction)

v(t): + | 4 | − | 12 | +

~~s(4) = 256 s(12) = 0~~

left (4,12)

right (0,4) (12,∞)

$a(t) = 6t - 48$

$0 = 6t - 48$

$0 = 6(t-8)$

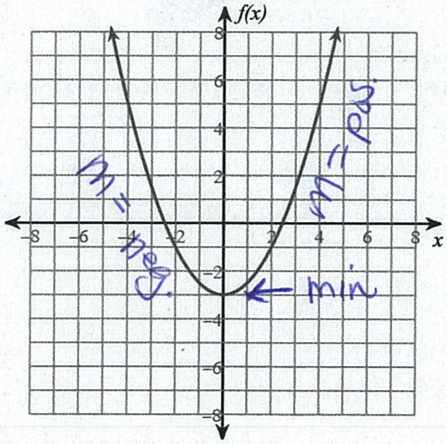
$t = 8$ sec

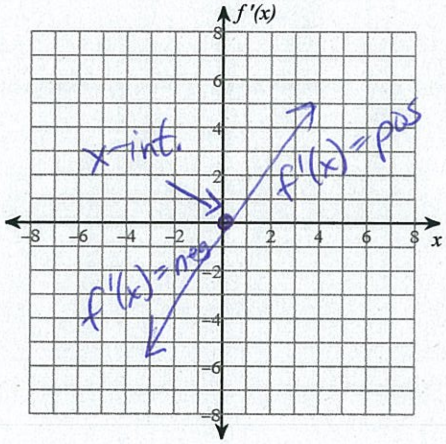
a(t): − | 8 | +

inc. (8,∞)

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

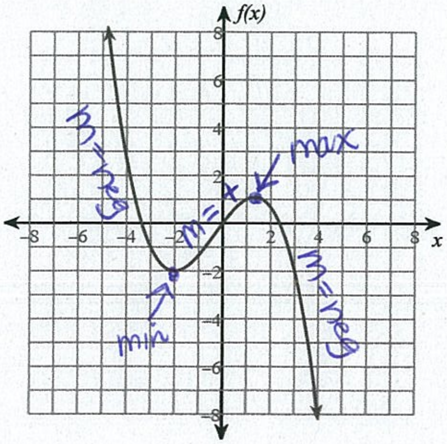
7)

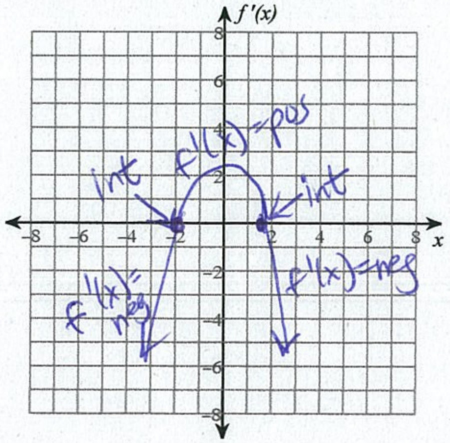




note: Actual slope of line may vary (but must be positive!)

8)



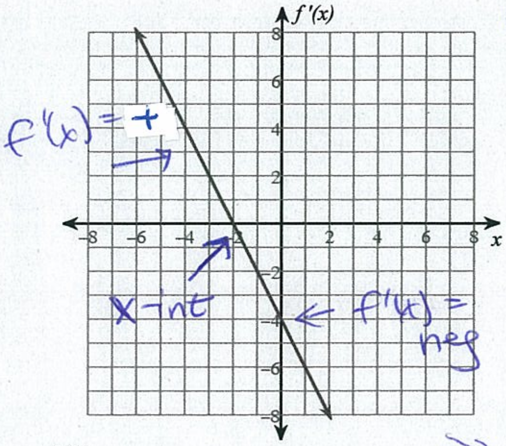


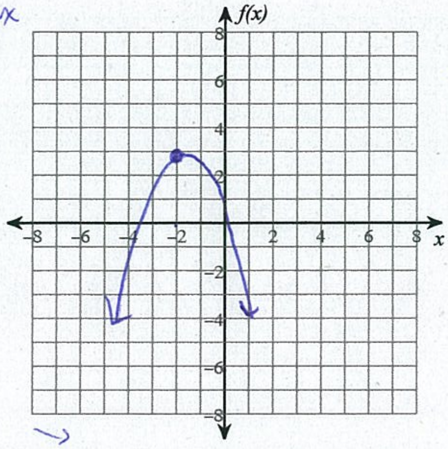
note: Actual y-value of max may vary!

Given the graph of $f'(x)$, sketch a possible graph of $f(x)$.

9)

↗ ↘
\+ | −
max



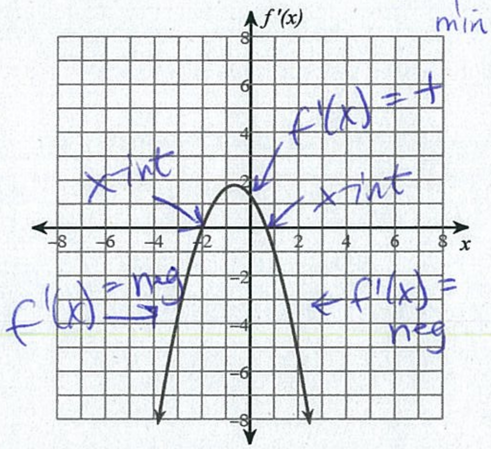


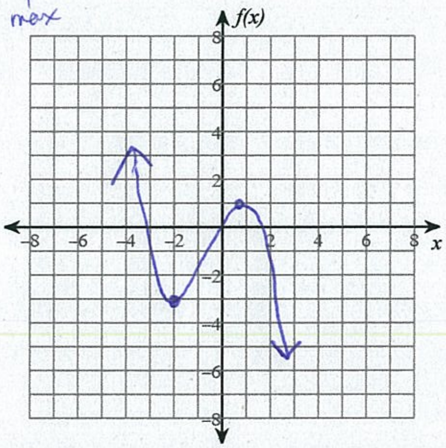
note: Actual y-value of max may vary.

Also, your x-intercepts may vary.

10)

↘ ↗ ↘
− | + | −
min max



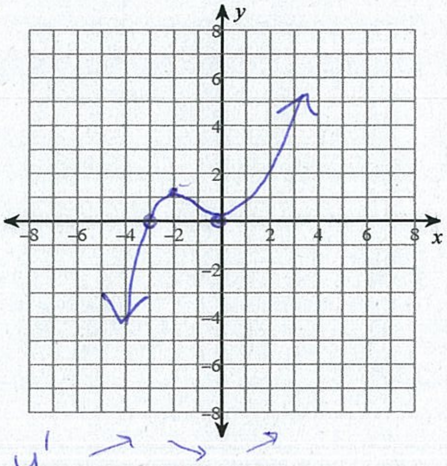


Note: Actual y-value of min/max may vary.

Also, your x-intercept may vary.

Find the x and y intercepts, open intervals where the function is increasing and decreasing, the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

11) $y = \frac{x^3}{3} + x^2$



y' ↗ ↘ ↗
\+ | − | +
−2 0
max min

x-int: $0 = \frac{x^3}{3} + x^2$

$0 = x^2\left(\frac{x}{3}+1\right)$

$x^2 = 0$ $\boxed{x=-3}$

$\boxed{x=0}$ ← touches

y-int: 0

$y' = x^2 + 2x$

$0 = x^2 + 2x$

$0 = x(x+2)$

$x = 0, x = -2$

$f(0) = 0$ $f(-2) = 4/3$

min $(0,0)$ max $(-2, 4/3)$

inc. $(-\infty, -2)(0, \infty)$

dec $(-2, 0)$

$y'' = 2x + 2$

$0 = 2x + 2$

$x = -1$

y' − | +
−1

$f(-1) = 2/3$

inf. pt $(-1, 2/3)$

cc↑ $(-1, \infty)$

cc↓ $(-\infty, -1)$