Ch 4 & 5 Misc Review Problems
SOLUTIONS:

3. (a) $a = 140$, $b = 98$

   (b) $C = 140 + (98)(5) = \$630$

   (c) $826 = 140 + 98x \implies x = 7$ hours

4. (a) i. Domain = $\{x \mid x \in \mathbb{R}\}$, Range = $\{y \mid y \geq -5\}$

      ii. x-intercept: -1, 5; y-intercept $= -25/9$

      iii. Function

   (b) i. Domain = $\{x \mid x \in \mathbb{R}\}$, Range = $\{y \mid y = 1$ or $-3\}$

      ii. x-intercept: none; y-intercept $= 1$

      iii. Function

6. (a) B   (b) C   (c) A   (d) D

5. (b) Since the parabola is symmetric, if 7 is an intercept (which is 3 units away from the axis of 4) then 1 is also an intercept (since it is 3 units away from 4). You also have a y-intercept of -2, which means a point of $(0, -2)$

$$y = a\,(x - \#)(x - \#)$$
$$y = a\,(x - 1)(x - 7)$$
$$-2 = a\,(0 - 1)(0 - 7)$$
$$-2 = 7a$$
$$a = -2/7$$
$$y = -2/7\,(x - 1)(x - 7)$$

7. The graph is symmetric around $x = 3$. So since there is an intercept at 1 (2 units away from $x = 3$) then there is also an intercept at $(5, 0)$. So $a = 5$

   Since the y-intercept is at $(0, 10)$ and that is 3 units away from the vertex, then the point $(b, 10)$ is also symmetric so it must also be 3 units away from the vertex. So the x-value is 3 units away from $(3, -8)$. So $b = 6$.

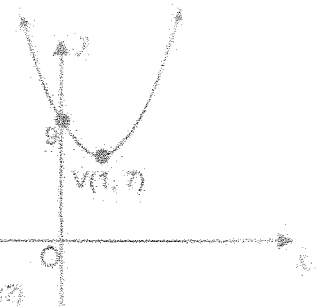**8.** **(a)** The mapping diagram shows a function f mapping members of a set X to members of set Y.
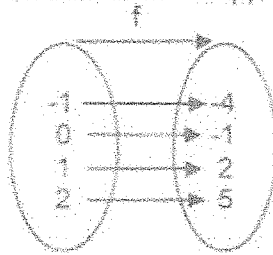
(i) Using set notation, write down the members of the domain and range.

$$D: \{x \mid x \in -1, 0, 1, 2\} \qquad R: \{y \mid y \in -4, -1, 2, 5\}$$

(ii) Find the equation of the function f.

$$y = 3x - 1$$

10. A graph of the quadratic $y = ax^2 + bx + c$ is shown alongside including the vertex, V, and the y-intercept.

(a) Determine the value of c.

$$9 = a(0)^2 + b(0) + c$$
$$9 = c$$

(b) Use the axis of symmetry to write an equation involving a and b.
(Hint: Think of the x-coordinate of the vertex...What is the formula for the x-coordinate of the vertex?)

$$x = -\frac{b}{2a} \qquad 1 = -\frac{b}{2a} \qquad 2a = -b \qquad \boxed{2a + b = 0}$$

(c) Use the point (1, 7) to write a second equation involving a and b.
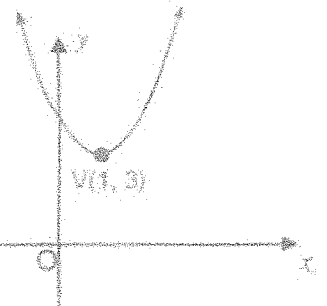
$$7 = a(1)^2 + b(1) + 9 \qquad \boxed{a + b = -2}$$
$$7 = a + b + 9$$

(d) Find a and b.

$$2a + b = 0$$
$$a + b = -2 \qquad \boxed{\begin{aligned} a &= 2 \\ b &= -4 \end{aligned}}$$

14. The diagram shows the graph of the quadratic function $f(x) = x^2 - mx + n$ including the vertex, V.

(a) Determine the values of m and n.

$$x = -\frac{b}{2a} \qquad 1 = \frac{-(-m)}{2(1)} \qquad \boxed{m = 2}$$

$$3 = (1)^2 - 2(1) + n$$
$$3 = 1 - 2 + n$$
$$3 = -1 + n \qquad \boxed{n = 4}$$

(b) Find k given that the graph passes through the point (3, k).

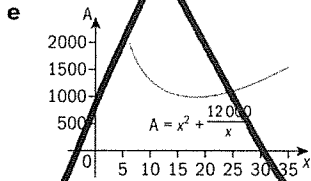$$k = (3)^2 - 2(3) + 4$$
$$k = 9 - 6 + 4 \qquad \boxed{k = 7}$$

(c) Find the domain and range of $f(x)$.

$$D: (-\infty, \infty)$$
$$R: [3, \infty)$$

8 a 3000 cm³

b $x^2 y = 3000$ $\therefore$ $y = \frac{3000}{x^2}$

c $A = x^2 + 4xy = x^2 + 4x \left(\frac{3000}{x^2}\right)$
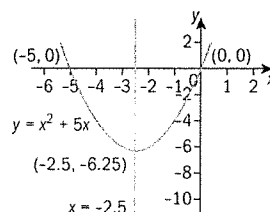
$A = x^2 + \frac{12000}{x}$

d

x(m)	5	10	15	20	25	30	35
$A(x)$ (cm²) (2sf)	2400	1300	1000	1000	1100	1300	1600

e

$A = x^2 + \frac{12000}{x}$

f $x = 18.2$

Review exercise

Paper 1 style questions

1 a 0000 – 0600

b 1130 – 1700

c 13° C

2 $c = nr + s$

a $35000 = 6r + s$

$116000 = 24r + s$

$18r = 81000$

$\therefore r = 4500$ SGD

b $35000 = 6 \times 4500 + s$

$s = 8000$ SGD

3 a $x^2 + 5x = x(x + 5)$

b

$(-5, 0)$ $(0, 0)$

$y = x^2 + 5x$

$(-2.5, -6.25)$

$x = -2.5$
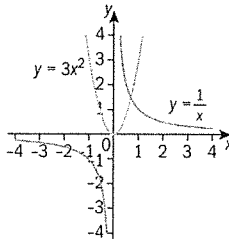
4 $h(t) = 30t - 5t^2$ $\quad 0 \le t \le 6$

a $h(4) = 40$ m

b 45 m

c from $t = 1$ to $t = 5$, $\therefore$ 4 s

5 a $f(x) = \frac{2^x}{m}$

$(3, 1.6)$ $\quad 1.6 = \frac{2^3}{m}$ $\quad \therefore m = \frac{8}{1.6} = 5$

b $f(x) = \frac{2^x}{5}$

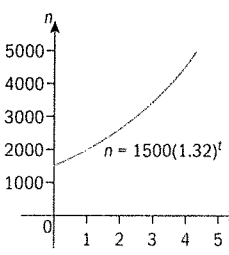$(0, n)$ $\quad n = \frac{1}{5}$

$f(2) = \frac{2^2}{5} = \frac{4}{5}$

6 a $x^2 - 2x - 15 = (x - 5)(x + 3)$

b i At A, $x = -3$ $\quad A = (-3, 0)$
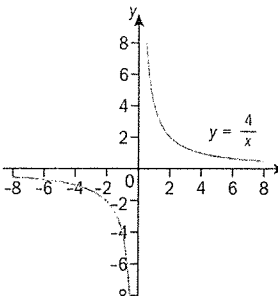
ii At B, $x = 1$ $\quad B = (1, -16)$

7 a ii

b i

c iii

d iv

8 a i $A(-1.68, 1.19)$

ii $B(2.41, -1.81)$

b $f(x) < g(x)$ $\quad -1.68 < x < 2.41$

c $y = -2$

9 a width $= 2.2 - x$

b $A = x(2.2 - x)$

c For maximum area, $x = 1.1$ m

10 a

$y = 3x^2$

$y = \frac{1}{x}$

b $x = 0, y = 0$

c $x = 0.693$

Paper 2 style questions

1 $n = 1500(1.32)^t$

a 1980, 4554

b

$n = 1500(1.32)^t$

c i $n = 1500(1.32)^{2.5} = 3000$

ii $t = 4.3366$ hours

$= 4$ hours 20 mins

2 a

$y = \frac{4}{x}$

b $y = 0, x = 0$

c
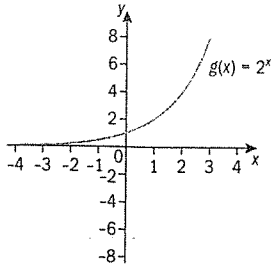
d $x = \pm 1.41$

e $\{y : y \in \mathbb{R}, y \neq 0\}$

3 $f(x) = 2(1.5)^x + 3$

   a $a = 4.33$ $b = 7.5$

   b
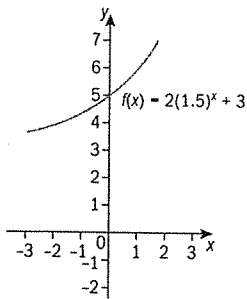
   c $f(x) > 3$

   d $x = 3.09$

   e $y = 3$

4 $f(t) = 21 + 77(0.8)^t$

   a $f(0) = 21 + 77 = 98°C$

   b $y = 21$

   c $21°C$

   d $f(8) = 33.9°C$

5 **a**
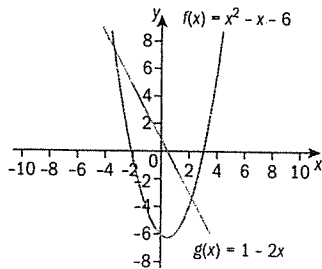
   b $(0.5, -6.25)$

   c -2

   d $(0, 1)$

   e $(2.19, -3.39)$,
      $(-3.19, 7.39)$

   f $x = 2.19, -3.19$

6 **a**
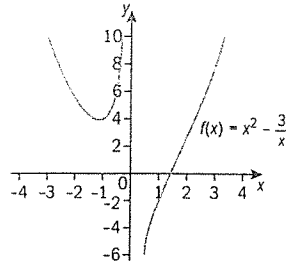
   b $x = 0$

   c
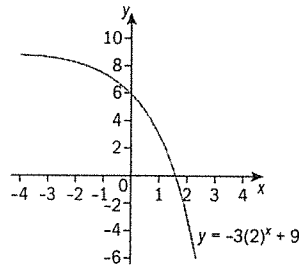
   d $y = 9$

   e $(-2.73, 8.55), (-0.454, 6.81), (1.53, 0.362)$

7 $P = \frac{-x^2}{10} + 10x - 60$

   a

x	0	10	20	30	40	50	60	70	80	90
P	-60	30	100	150	180	190	180	150	100	30

   b
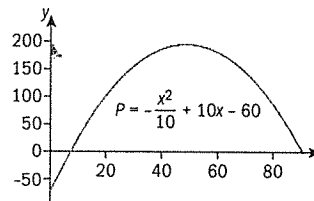
   c **i** 190 euros   **ii** 50   **iii** 33 or 67
      iv 60 euros

8 **a**
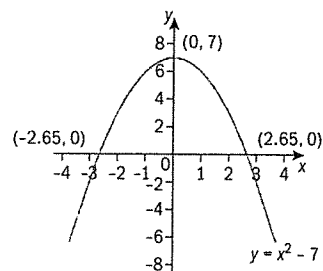

   $(0, -7), (2.65, 0), (-2.65, 0)$

   b

   c $x = \pm 2.65$