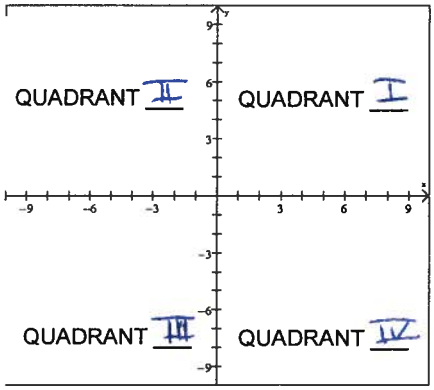
CARTESIAN PLANE: A Cartesian plane is made up of a two dimensional flat surface that is divided into 4 quadrants. These quadrants are the result of a plane divided by two straight lines intersecting at right angles and meeting at a point called the origin.



The horizontal axis is called the x - axis.

The vertical axis is called the y - axis

DISTANCE FORMULA: To find the distance between two points on a plane, given $P(x_1, y_1)$ and $Q(x_2, y_2)$ you use the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

MIDPOINT FORMULA: To find the midpoint of a line segment connecting two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ you use the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

in formula booklet

Example: Find the value of b, given that A(-4, 3) and B(b, 7) are $\sqrt{97}$ units apart. Distance!

$\sqrt{(b-(-4))^2 + (7-3)^2} = \sqrt{97}$

$(b+4)^2 + 4^2 = 97$

$(b+4)^2 = 97 - 16$

$(b+4)^2 = 81$

$b+4 = \pm 9$

$b+4=9$ $\quad b=5$ or $b+4=-9$ $\quad b=-13$

must have both!

SLOPE (GRADIENT) FORMULA: To find the slope of a line passing through two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ you use the formula:

in formula booklet → $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$ **NOTE: SLOPE = GRADIENT IN IB TERMINOLOGY!!**

Example: Find the gradient of the line passing through the points (4, 3) and (-2, -5).

$\frac{-5-3}{-2-4} = \frac{-8}{-6} = \frac{4}{3}$ ← leave as a reduced fraction!

FORMS OF LINEAR EQUATIONS:

GRADIENT-INTERCEPT (Slope-Intercept) FORM: $y = mx + c$ where **m** is the gradient and **c** is the y-intercept → says equation of straight line

in formula booklet

STANDARD FORM: $ax + by + d = 0$ where $m = -\frac{a}{b}$ y-intercept $= -\frac{d}{b}$ x-intercept $= -\frac{d}{a}$ ← not in booklet

POINT-GRADIENT FORM: $y - y_1 = m(x - x_1)$ where **m** is the gradient & (x_1, y_1) is a point on the line.

Example: Write the equation of the line that passes through the points (4, 3) and (-2, -5) in point-gradient form, gradient-intercept form, and standard form.

Find slope first

$\frac{-5-3}{-2-4} = \frac{4}{3}$ (from above)

point gradient ... pick a pt

either ...

$y - 3 = \frac{4}{3}(x - 4)$ or $y - (-5) = \frac{4}{3}(x - (-2))$ or $y + 5 = \frac{4}{3}(x + 2)$

Gradient-intercept

either distribute last answer or plug in slope + a pt + solve for c

$y = mx + c$

$3 = \frac{4}{3}(4) + c$

$3 = \frac{16}{3} + c$

$\frac{9}{3} = \frac{16}{3} + c$

$-\frac{7}{3} = c$

$y = \frac{4}{3}x - \frac{7}{3}$

Standard form

move last answer around + get rid of fractions

$3y = 4x - 7$

$0 = 4x - 3y - 7$

(A should be positive)

PARALLEL LINES:
Two straight lines are parallel if they have the same gradient.
Conversely, two lines with the same gradient are parallel lines.

$\ell_1 \parallel \ell_2$ iff $m_1 = m_2$

PERPENDICULAR LINES:
If two lines are perpendicular, then the product of their gradients is -1. (gradients are ~~negative~~ opposite reciprocals)

$\ell_1 \perp \ell_2$ iff $m_1 \bullet m_2 = -1$ or $m_1 = -\frac{1}{m_2}$

Example: Determine whether the two lines given are parallel, perpendicular, or neither. Put in slope int. form

ℓ_1: $2x - y - 5 = 0$ $2x-5=y$ $m=2$
ℓ_2: $4x - 2y + 3 = 0$ $4x+3=2y$ $2x+3/2=y$ $m=2$

parallel slopes are equal

ℓ_1: $3x - 4y + 28 = 0$ $3x+28=4y$ $3/4x+7=y$
ℓ_2: $4x + 3y - 6 = 0$ $3y=-4x+6$ $y=-\frac{4}{3}x+2$
$m=3/4$ $m=-\frac{4}{3}$ ← reciprocals and opposite!
perpendicular

PERPENDICULAR BISECTOR: The perpendicular bisector of a line segment $\overline{AB}$ is the line that passes through the midpoint M of $\overline{AB}$ and is perpendicular to it.

Example: Find the equation of the perpendicular bisector of the line segment $\overline{AB}$, where A = (1, 2) and B = (3, 1). Put your answer in ax + by + **d** = 0 form.

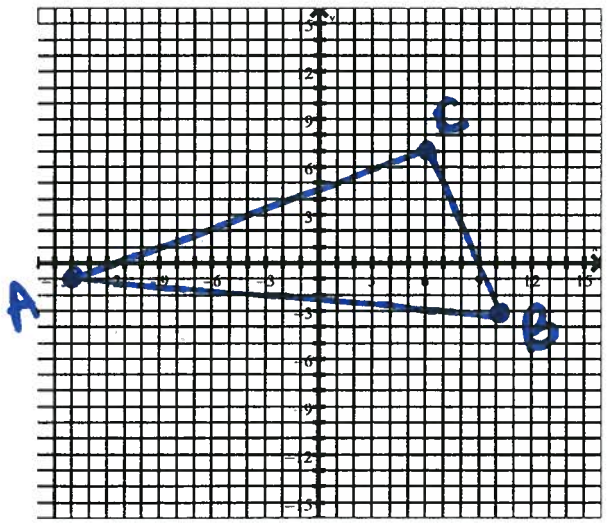
find m
$\left(\frac{1+3}{2}, \frac{2+1}{2}\right)$
$\left(\frac{4}{2}, \frac{3}{2}\right) = (2, 3/2)$

Slope of AB
$\frac{1-2}{3-1} = \frac{-1}{2}$

⊥ slope
$m=2$

Find pt through m with m=2
$y-3/2 = 2(x-2)$
$y-3/2 = 2x-4$
$2y-3 = 4x-8$
$0 = 4x-2y-5$

Example: Show that triangle ABC, where A is (-14, -1) B is (10, -3) and C is (6, 7) is a **right** triangle, by finding the slopes of the line segments $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. Graph the triangle .



To be a right triangle
I must have two perp. sides
It looks like it would be at C.

m of AC
$\frac{7-(-1)}{6-(-14)} = \frac{7+1}{6+14} = \frac{8}{20} = \frac{2}{5}$

m of BC
$\frac{7-(-3)}{6-10} = \frac{7+3}{6-10} = \frac{10}{-4} = \frac{5}{-2}$

Since $\frac{2}{5}$ & $-\frac{5}{2}$ are perp slopes
AC + BC form a rt angle

thus it is a right triangle!

POINTS ON LINES AND INTERCEPTS:

A POINT ON A LINE: If a point is on a line, when you plug it into the equation of the line, you will get a true statement.

Example: Find t if $(3, t)$ lies on the line with equation $4x + 5y = -1$

$x=3$, $y=t$ $\quad 4(3)+5t=-1$ $\quad 12+5t=-1$ $\quad 5t=-13$ $\quad t=-13/5$

INTERCEPTS: Intercepts are where a line cuts off the x or y axis.

Example: Find where the line $5x - 3y - 12 = 0$ cuts off the axes (basically... find the intercepts)

x-int: $5x-3(0)-12=0$ $\quad 5x-12=0$ $\quad 5x=12$ $\quad x=12/5$ $\quad (12/5, 0)$

y-int: $5(0)-3y-12=0$ $\quad -3y-12=0$ $\quad -3y=12$ $\quad y=-4$ $\quad (0,-4)$

WRITING EQUATIONS OF LINES:

Example: Write the equation of the line in standard form $ax + by + d = 0$, given that the gradient is 5 and the y-intercept is -4.

$c=-4$, $m=5$ $\quad y=mx+c$ $\quad y=5x-4$ $\quad 0=5x-y-4$

Example: Write the equation of the line in point slope form $y - y_1 = m(x - x_1)$ given that the line has a gradient of 3 and passes through the point (2, -1).

$m=3$, $pt(2,-1)$ $\quad y-(-1)=3(x-2)$ $\quad y+1=3(x-2)$ $\quad y+1=3x-6$ $\quad y=3x-7$ $\quad 0=3x-y-7$

Example: Write the equation of the line in gradient-intercept form $y = mx + c$ given that the line passes through the points (2, -5) and (7, -6).

Find slope first: $\frac{-6-(-5)}{7-2}=\frac{-6+5}{7-2}$ $\quad m=-\frac{1}{5}$

pick a pt and either... $\quad y-(-5)=-\frac{1}{5}(x-2)$ $\quad y+5=-\frac{1}{5}x+\frac{2}{5}$ $\quad y=-\frac{1}{5}x+\frac{2}{5}-\frac{25}{5}$ $\quad y=-\frac{1}{5}x-\frac{23}{5}$

or $\quad -5=-\frac{1}{5}(2)+c$ $\quad -5=-\frac{2}{5}+c$ $\quad -\frac{25}{5}+\frac{2}{5}=c$ $\quad c=-\frac{23}{5}$ $\quad y=-\frac{1}{5}x-\frac{23}{5}$

Example: Write the equation of the line that is perpendicular to the line $3x - 2y - 18 = 0$ and passes through the point (-9, 3). Put your answer in standard form $ax + by + c = 0$.

first find slope: $3x-2y-18=0$ $\quad 3x-18=2y$ $\quad 3/2\,x-9=y$ $\quad m=3/2$ so $\perp m=-2/3$

$m=-2/3$ $\quad pt(-9,3)$ $\quad y-3=-2/3(x+9)$ $\quad y-3=-\frac{2}{3}x-\frac{18}{3}$ $\quad y-3=-\frac{2}{3}x-6$ $\quad y=-2/3x-3$ $\quad 3y=-2x-9$ $\quad 2x+3y+9=0$

Example: The lines $px - 6y + 6 = 0$ and $2x + y + p = 0$ are perpendicular. Find the value of p.

$px-6y+6=0$ $\quad px+6=6y$ $\quad p/6\,x+1=y$ $\quad m=p/6$

$2x+y+p=0$ $\quad y=-2x+p$ $\quad m=-2$

Since they are perp if the 2nd equation has $m=-2$ then the slope of the 1st equation is $1/2$

$p/6=1/2$ $\quad p=6(\frac{1}{2})$ $\quad p=3$