

15

Probability distributions

Answers

Skills check

1 a $\bar{x} = \frac{\sum fx}{\sum f} = \frac{(3 \times 3) + (4 \times 5) + (5 \times 7) + (6 \times 9) + (7 \times 6) + (8 \times 2)}{3 + 5 + 7 + 9 + 6 + 2}$
 $= \frac{176}{32} = 5.5$

b $\bar{x} = \frac{(10 \times 3) + (12 \times 10) + (15 \times 15) + (17 \times 9) + (20 \times 2)}{3 + 10 + 15 + 9 + 2}$
 $= \frac{568}{39} = 14.6$ (3 sf)

2 a $\binom{6}{2} = \frac{6!}{2!4!} = \frac{6 \times 5}{2} = 15$

b $\binom{8}{5} = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

c $\binom{9}{6} (0.3)^3 (0.7)^6 = 0.267$

3 a $\frac{5.5}{x} = 3.2 \therefore 5.5 = 3.2x \therefore x = \frac{5.5}{3.2} = 1.71875$

b $\frac{x-2.5}{1.2} = 0.4 \therefore x - 2.5 = 0.48 \therefore x = 2.98$

c $\frac{9-x}{0.2} = 1.6 \therefore 9 - x = 0.32 \therefore x = 9 - 0.32 = 8.68$

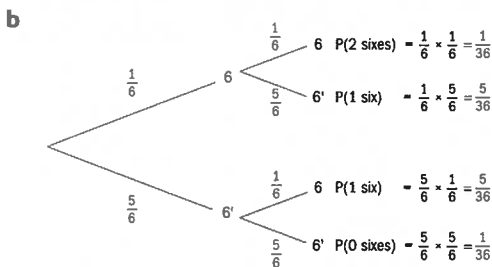
Exercise 15A

- 1 a discrete
 b continuous
 c discrete
 b continuous

2 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

s	2	3	4	5	6	7	8	9	10	11	12
P(S = s)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



P(2 sixes) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

P(1 six) = $\frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$

P(1 six) = $\frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

P(0 sixes) = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

n	0	1	2
P(N = n)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

c

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

n	1	2	3	4	5	6
P(N = n)	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

d

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	8	9	12	15	18
4	4	10	12	16	20	24
5	5	12	15	20	25	30
6	6	14	18	24	30	36

p	1	2	3	4	5	6	8	9	10
P(P = p)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$
p	12	15	16	18	20	24	25	30	36
P(P = p)	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

- 3 a The faces are numbered 1, 2, 2, 3, 3, 3

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6



t	2	3	4	5	6
P(T = t)	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

b $P(T > 4) = \frac{12}{36} + \frac{9}{36} = \frac{21}{36} = \frac{7}{12}$

4 a

no. on dice	1	2	3	4	5	6
s	2	1	6	2	10	3

s	1	2	3	6	10
P(S = s)	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

b $P(S > 2) = \frac{3}{6} = \frac{1}{2}$

5 a $\frac{1}{3} + \frac{1}{3} + c + c = 1$

$2c = \frac{1}{3} \therefore c = \frac{1}{6}$

b $P(1 < X < 4) = P(X = 2) + P(X = 3) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

6 $P(Y = y) = cy^3 \quad y = 1, 2, 3$

y	1	2	3
P(Y = y)	c	8c	27c

$36c = 1 \therefore c = \frac{1}{36}$

7 $2k + 4k^2 + 6k^2 + k = 1$

$10k^2 + 3k - 1 = 0$

$(5k - 1)(2k + 1) = 0$

$k = \frac{1}{5}$ (k cannot be negative)

8 $P(X = x) = k\left(\frac{1}{3}\right)^{x-1} \quad x = 1, 2, 3, 4$

x	1	2	3	4
P(X = x)	k	$\frac{1}{3}k$	$\frac{1}{9}k$	$\frac{1}{27}k$

$k + \frac{1}{3}k + \frac{1}{9}k + \frac{1}{27}k = 1$

$\frac{40}{27}k = 1 \therefore k = \frac{27}{40}$

9 a

x	0	1	2	3	4	5
P(X = x)	a	a	a	b	b	b

$3a + 3b = 1 \quad (1) \quad P(X \geq 2) = 3P(X < 2)$

$a + 3b = 6a \rightarrow 3(2a)$

$3b = 5a \quad (2)$

substitute (2) and (1) $3a + 5a = 1$

$a = \frac{1}{8} \quad b = \frac{5}{24}$

b $P(5, 3) = \frac{5}{24} \times \frac{5}{24} = \frac{25}{576} \quad P(3, 5) = \frac{25}{576}$

$P(5, 4) = \frac{25}{576} \quad P(4, 5) = \frac{25}{576} \quad P(5, 5) = \frac{25}{576}$

$P(\text{sum} > 7) = \frac{125}{576}$

10 a $P(C = 3) = P(A = 1 \text{ and } B = 2) + P(A = 2 \text{ and } B = 1)$
 $= \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{6} = \frac{2}{9} + \frac{1}{18} = \frac{5}{18}$

b $P(C = 2) = P(A = 1 \text{ and } B = 1) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
 $P(C = 4) = P(A = 1 \text{ and } B = 3) + P(A = 2 \text{ and } B = 2) + P(A = 3 \text{ and } B = 1)$

$= \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{6} = \frac{6}{18}$

$P(C = 5) = P(A = 2 \text{ and } B = 3) + P(A = 3 \text{ and } B = 2)$

$= \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{2}{3} = \frac{5}{18}$

$P(C = 6) = P(A = 3 \text{ and } B = 3) = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$

c	2	3	4	5	6
P(C = c)	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{6}{18}$	$\frac{5}{18}$	$\frac{1}{18}$

Investigation – dice scores

1

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

d

	0	1	2	3	4	5
P(D = d)	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

2

d	0	1	2	3	4	5
Expected frequency	6	10	8	6	4	2

3 Mean = $\frac{(0 \times 6) + (1 \times 10) + (2 \times 8) + (3 \times 6) + (4 \times 4) + (5 \times 2)}{36}$
 $= \frac{70}{36} = \frac{35}{18}$

4

d	0	1	2	3	4	5
Expected frequency	$\frac{150}{9}$	$\frac{250}{9}$	$\frac{200}{9}$	$\frac{150}{9}$	$\frac{100}{9}$	$\frac{50}{9}$

Mean = $\frac{(0 \times \frac{150}{9}) + (1 \times \frac{250}{9}) + (2 \times \frac{200}{9}) + (3 \times \frac{150}{9}) + (4 \times \frac{100}{9}) + (5 \times \frac{50}{9})}{100}$
 $= \frac{35}{18}$

5 The means are the same

6 $\frac{35}{18}$

Exercise 15B

x	1	4	9	16	24	36
P(X = x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \left(1 \times \frac{1}{6}\right) + \left(4 \times \frac{1}{6}\right) + \left(9 \times \frac{1}{6}\right) + \left(16 \times \frac{1}{6}\right) + \left(25 \times \frac{1}{6}\right) + \left(36 \times \frac{1}{6}\right)$$

$$= \frac{91}{6} = 15.2 \text{ (3 sf)}$$

2 $\frac{3}{6} + x + y = 1 \quad \therefore x + y = \frac{1}{2} \quad (1)$

$$E(Z) = 5 \frac{2}{3} \therefore$$

$$\left(2 \times \frac{1}{6}\right) + \left(3 \times \frac{1}{6}\right) + \left(5 \times \frac{1}{6}\right) + 7x + 11y = \frac{17}{3}$$

$$7x + 11y = 4 \quad (2)$$

Solving (1) and (2), $x = \frac{3}{8}, \quad y = \frac{1}{8}$

x	1	2	3	5	8	13
P(X = x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(X) = \frac{1}{6}(1 + 2 + 3 + 5 + 8 + 13) = \frac{16}{3} \text{ or } 5\frac{1}{3}$$

x	1	2	3	4	5	6	7	8
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{7}{36}$	$\frac{8}{36}$

$$E(X) = \frac{1}{36} + \frac{4}{36} + \frac{9}{36} + \frac{16}{36} + \frac{25}{36} + \frac{36}{36} + \frac{49}{36} + \frac{64}{36}$$

$$= \frac{204}{36} = 5\frac{2}{3}$$

x	1	2	3	4	5	6	7	8	9
P(X = x)	k	2k	3k	4k	5k	4k	3k	2k	k

$$25k = 1 \quad \therefore k = \frac{1}{25}$$

b From symmetry, $E(X) = 5$

x	1	2	3
P(X = x)	0.2	$1 - k$	$k - 0.2$

b $0 \leq 1 - k \leq 1 \quad \text{and} \quad 0 \leq k - 0.2 \leq 1$

$$-1 \leq -k \leq 0 \quad 0.2 \leq k \leq 1.2$$

$$1 \geq k \geq 0$$

$$\therefore 0.2 \leq k \leq 1$$

c Mean = $0.2 + 2(1 - k) + 3(k - 0.2)$

$$= 0.2 + 2 - 2k + 3k - 0.6 = k + 1.6$$

x	1	2	4
P(x = x)	a	0.3	b

$$a + b = 0.7 \quad (1)$$

$$\text{mean} = 2.8 \quad \therefore a + 0.6 + 4b = 2.8$$

$$a + 4b = 2.2 \quad (2)$$

solving (1) and (2), $a = 0.2, b = 0.5$

$$\therefore P(x = 1) = 0.2$$

8 a $P(R = 1) = \frac{2}{10} = \frac{1}{5}$

$$P(R = 2) = \frac{8}{10} \times \frac{2}{9} = \frac{16}{90} = \frac{8}{45}$$

$$P(R = 3) = \frac{8}{10} \times \frac{7}{9} \times \frac{2}{8} = \frac{14}{90} = \frac{7}{45}$$

$$P(R = 4) = \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{2}{7} = \frac{12}{90} = \frac{6}{45}$$

$$P(R = 5) = \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times \frac{2}{6} = \frac{10}{90} = \frac{5}{45}$$

$$P(R = 6) = \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} \times \frac{2}{5} = \frac{8}{90} = \frac{4}{45}$$

$$P(R = 7) = \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{6}{90} = \frac{3}{45}$$

$$P(R = 8) = \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{4}{90} = \frac{2}{45}$$

$$P(R = 9) = \frac{8}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$$

$$= \frac{2}{90} = \frac{1}{45}$$

r	1	2	3	4	5	6	7	8	9
P(R = r)	$\frac{9}{45}$	$\frac{8}{45}$	$\frac{7}{45}$	$\frac{6}{45}$	$\frac{5}{45}$	$\frac{4}{45}$	$\frac{3}{45}$	$\frac{2}{45}$	$\frac{1}{45}$

b Mean = $\frac{1}{45}(9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 9) = 3\frac{2}{3}$

9 a $P(R = 2) = \frac{8}{10} \times \frac{2}{10} = \frac{16}{100} = \frac{4}{25}$

b $P(R = 3) = \left(\frac{8}{10}\right)^2 \times \frac{2}{10} = \frac{16}{125}$

c $P(R = n) = 0.8^{n-1} \times 0.2$

d 1

10 a $P(Z = 0) + 0.2 + 0.05 + 0.001 + 0.0001 = 1$

$$P(Z = 0) = 1 - 0.2511$$

$$= 0.7489$$

b $E(Z) = 0 + 0.4 + 1 + 0.2 + 0.1$

$$= 1.7$$

\$1.70 is the expected winnings on a ticket

c A ticket costs \$2, but you only expect to win \$1.70. Therefore you expect to lose \$0.30

Investigation – the binomial quiz

You would expect to get 2.5 questions right

$$P(3 \text{ right}) = \binom{5}{3} (0.5)^3 (0.5)^2 = 0.3125$$

Exercise 15C

1 $X \sim B\left(4, \frac{1}{2}\right)$

a $P(X = 1) = \binom{4}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 = \frac{1}{4}$

b $P(X < 1) = P(X = 0) = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

c $P(X \leq 1) = P(X = 0 \text{ or } 1) = \frac{1}{16} + \frac{1}{4} = \frac{5}{16}$

d $P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{1}{16} = \frac{15}{16}$

- 2 using a GDC:
 a 0.329 b 0.351
 c 0.680 d 0.649
- 3 using a GDC:
 a $P(X = 5) = 0.0389$
 b $P(X < 5) = 0.952$
 c $P(X > 5) = 0.00870$
 d $P(X \geq 1) = 0.932$

↙ @ binompdf
 (b) binomcdf
 (c) a+b
 ↖ (d) 1-b

Exercise 15 D

(Using a GDC where possible)

- 1 $X \sim B(4, 0.25)$

x	0	1	2	3	4
$P(X=x)$	0.316	0.422	0.211	0.0469	0.00391

Most likely outcome is 1 red face with probability 0.422

- 2 $X \sim B(8, 0.55)$
 a $P(X = 5) = 0.257$
 b $P(\text{at least 5 times}) = P(X \leq 3) = 0.260$
- 3 $X \sim B(16, 0.01)$
 a $P(X = 0) = 0.851$
 b $P(13 \text{ not faulty}) = P(X = 3) = 0.000491$
 c $P(X \geq 2) = 0.0109$
- 4 $X \sim B(10, 0.25)$
 a $P(X = 5) = 0.0584$
 b $P(\text{at least 3 free}) = P(X \leq 7) = 0.9996$
- 5 $X \sim B(5, 0.4)$
 $P(X \leq 3) = 0.913$
- 6 $X \sim B(6, 0.15)$
 a $P(X > 1) = 0.224$
 b $P(X = 1) = 0.399$
- 7 $X \sim B(15, 0.05)$
 a i $P(X = 3) = 0.0307$
 ii $P(X = 0) = 0.463$
 iii $P(X \geq 2) = 0.171$
 b i $(0.46329\dots)^2 = 0.215$
 ii $(0.17095\dots)^2 = 0.0292$
 iii $0.46329\dots \times 0.17095\dots \times 2 = 0.158$

Exercise 15 E

- 1 $X \sim B(n, 0.6)$ $P(X < 1) = 0.0256$
 $P(X = 0) = 0.0256$
 $(0.4)^n = 0.0256$
 $n \log 0.4 = \log 0.0256$
 $n = 4$

- 2 $X \sim B(n, 0.01)$ $P(x = 0) > 0.5$
 $0.99^n > 0.5$
 $n \log 0.99 > \log 0.5$
 $n < \frac{\log 0.5}{\log 0.99} = 68.9$, so the largest
 sample size is 68

- 3 $X \sim B(n, 0.2)$ $P(X \geq 1) > 0.75$
 $1 - P(X = 0) > 0.75$
 $1 - 0.8^n > 0.75$
 $0.25 > 0.8^n$
 $0.8^n < 0.25$
 $n \log 0.8 < \log 0.25$
 $n > \frac{\log 0.25}{\log 0.8}$
 $n > 6.21$
 \therefore least value of $n = 7$

- 4 $X \sim B(n, 0.3)$ $P(x \geq 1) > 0.95$
 $1 - P(x = 0) > 0.95$
 $1 - 0.7^n > 0.95$
 $0.05 > 0.7^n$
 $0.7^n < 0.05$
 $n \log 0.7 < \log 0.05$
 $n > \frac{\log 0.05}{\log 0.7}$
 $n > 8.399$
 \therefore least number of attempts is 9

- 5 $X \sim B(n, 0.5)$ $P(X \geq 1) > 0.99$
 $1 - P(X = 0) \geq 0.99$
 $1 - 0.5^n \geq 0.99$
 $0.01 \geq 0.5^n$
 $0.5^n \leq 0.01$
 $n \log 0.5 \leq \log 0.01$
 $n \geq \frac{\log 0.01}{\log 0.5}$
 $n > 6.64$ so the coin must be
 tossed 7 times.

Exercise 15 F

- 1 a $X \sim B(40, 0.5)$ $E(X) = 40 \times 0.5 = 20$
 b $X \sim B\left(40, \frac{1}{6}\right)$ $E(X) = 40 \times \frac{1}{6} = 6\frac{2}{3}$
 c $X \sim B(40, 0.25)$ $E(X) = 40 \times 0.25 = 10$
- 2 $X \sim B(n, p)$ mean = 10 $p = 0.4$ $np = 10$
 $n \times 0.4 = 10$ $\therefore n = 25$
- 3 a $X \sim B(15, 0.25)$
 b mean = $15 \times 0.25 = 3.75$
 c $P(X \geq 10) = 0.000795$

- 4 a total number of girls =
 $(1 \times 34) + (2 \times 40) + (3 \times 13) = 153$
 total number of children = $100 \times 3 = 300$
 $\therefore P(\text{girl}) = \frac{153}{300} = 0.51$
- b $X \sim B(3, 0.51)$ $P(x = 2) = 0.382$
 expected number of families = $0.382 \times 100 = 38.2$

Exercise 15G

- 1 $X \sim B(0, \frac{1}{4})$
 Mean = $0 \times \frac{1}{4} = 0$
 Variance = $0 \times \frac{1}{4} \times (1 - \frac{1}{4}) = 0$
- 2 $B(12, 0.6)$
 Mean = $12 \times 0.6 = 7.2$
 Variance = $12 \times 0.6 \times 0.4 = 2.88$
 Standard deviation = $\sqrt{2.88} = 1.70$ (3 sf)
- 3 $X \sim B(40, \frac{1}{2})$
 Mean = $40 \times \frac{1}{2} = 20$
 Variance = $40 \times \frac{1}{2} \times \frac{1}{2} = 10$
 Standard deviation = $\sqrt{10} = 3.16$ (3 sf)
- 4 $X \sim B(10, \frac{1}{6})$
- a $E(X) = 10 \times \frac{1}{6} = \frac{5}{3}$
- b $\text{Var}(X) = 10 \times \frac{1}{6} \times \frac{5}{6} = \frac{25}{18}$
- c $P(X < \mu) = P(X < \frac{5}{3}) = 0.485$ (3 sf)
 (using binomial CDF on the GDC)
- 5 $X \sim B(22, \frac{1}{5})$
- a $E(X) = 22 \times \frac{1}{5} = \frac{22}{5}$
- b $\text{Var}(X) = 22 \times \frac{1}{5} \times \frac{4}{5} = \frac{88}{25}$
- c $P(X < 4) = 0.332$ (3 sf) using binomial CDF on the GDC
- 6 $X \sim B(n, p)$
 $E(X) = 4.5$, $\text{Var}(X) = 3.15$
 $E(X) = np = 4.5$
 $\text{Var}(X) = npq = np(1 - p) = 3.15$
 $4.5(1 - p) = 3.15$
 $1 - p = \frac{3.15}{4.5}$
 $= 0.7$
 $p = 1 - 0.7$
 $= 0.3$
- $np = 4.5$
 $n = \frac{4.5}{p} = \frac{4.5}{0.3} = 15$
 $P(X \geq 3) = 1 - P(X < 3) = 1 - 0.126828 = 0.873$ (3 sf)

- 7 $X \sim B(n, p)$
 $E(X) = 7.8$
 $p = 0.3$
- a $E(X) = np = 7.8$
 $0.3n = 7.8$
 $n = \frac{7.8}{0.3}$
 $n = 26$
- b $\text{Var}(X) = npq = np(1 - p)$
 $= 26 \times 0.3 \times 0.7$
 $= 5.46$

- 8 $X \sim B(n, p)$
 $E(X) = 9.6$
 $\text{Var}(X) = 1.92$
 $E(X) = np = 9.6$
 $\text{Var}(X) = npq = np(1 - p) = 1.92$
 $9.6(1 - p) = 1.92$
 $9.6 - 9.6p = 1.92$
 $9.6p = 7.68$
 $p = 0.8$
 $n = \frac{9.6}{0.8}$
 $n = 12$

$P(X = 6) = 0.0155$ (using binomial PDF on the GDC)

Exercise 15H

Using GDC:

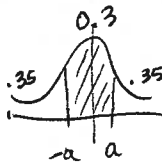
- 1 a 0.683
 b 0.954
 c 0.997
- 2 a $P(1 < Z < 2) + P(-2 < Z < -1) = 0.272$
 b $P(0.5 < Z < 1.5) + P(-1.5 < Z < -0.5) = 0.483$
- 3 a $P(Z > 1) = 0.159$
 b $P(Z > 2.4) = 0.00820$
- 4 a $P(Z < -1) = 0.159$
 b $P(Z < -1.75) = 0.0401$
- 5 a 0.742
 b 0.236
 c 0.0359
 d 0.977
 e 0.390
- 6 a 0.306
 b 0.595
 c 0.285

- 7 a $P(|Z| < 0.4) = P(-0.4 < Z < 0.4) = 0.311$
 b $P(|Z| > 1.24) = 1 - P(|Z| < 1.24)$
 $= 1 - P(-1.24 < Z < 1.24)$
 $= 0.215$

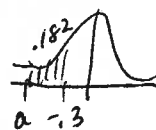
Exercise 15I

Using GDC:

- 1 a 0.655
 b 0.841
 c 0.186
 d 0.5
 2 a 0.672
 b 0.748
 c 0.345
 3 a 0.994
 b 0.997
 c 0.494



- 3 a $P(-a < Z < a) = 0.3$
 $\therefore P(Z < a) = 0.65 \therefore a = 0.385$
 b $P(|Z| > a) = 0.1096$
 $\therefore P(Z < a) = 0.9452 \therefore a = 1.60$
 4 a $P(Z < a) = 0.95 \therefore a = 1.64$
 b $P(Z < a) = 0.8 \therefore a = 0.842$

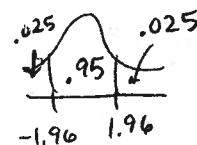


Exercise 15J

- 1 $X \sim N(100, 20^2)$
 a $P(X < 130) = 0.933$
 b $P(X > 90) = 0.691$
 c $P(80 < X < 125) = 0.736$
 2 $X \sim N(4, 0.25^2)$
 $P(3.5 < X < 4.5) = 0.9545$
 number of acceptable bolts = $0.9545 \times 500 = 477$
 3 $X \sim N(14, 4^2)$
 a $P(X > 20) = 0.0668$
 b $P(X > 10) = 0.159 = 15.9\%$
 4 $X \sim N(551.3, 15^2)$
 $P(X > 550) = 0.535 = 53.6\%$
 5 $X \sim N(500, 20^2)$
 a $P(X < 475) = 0.106$
 b $(0.1056 \dots)^3 = 0.00118$

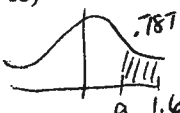
Exercise 15L

- 1 $X \sim N(5.5, 0.2^2)$ $P(X > a) = 0.235$
 $P(x < a) = 0.765 \therefore a = 5.64$
 2 $M \sim N(420, 10^2)$
 a $P(M < a) = 0.25 \therefore a = 413$
 b $P(M < b) = 0.9 \therefore b = 433$
 3 $X \sim N(502, 1.6^2)$
 a $P(x < 500) = 0.106$
 b $P(500 < x < 505) = 0.864$ or 86.4%
 c $P(x < b) = 0.975$ $b = 505.1$ $a = 498.9$
 $a = 499$ $b = 505$ $z = \frac{x - 502}{1.6}$
 4 $X \sim N(550, 25^2)$
 a $P(520 < X < 570) = 0.673$
 b $P(X > a) = 0.1 \therefore P(X < a) = 0.9 \therefore a = 582$
 5 a $X \sim N(55, 15^2)$, $P(x > d) = 0.05$, $d = 79.7$
 b $P(x < f) = 0.90$, $f = 35.8$



Exercise 15K

- 1 a $P(Z < a) = 0.922$, $a = 1.42$
 b $P(Z > a) = 0.342$
 $\therefore P(Z < a) = 0.658$, $a = 0.407$
 c $P(Z > a) = 0.005 \therefore P(Z < a) = 0.995$,
 $a = 2.58$
 2 a $P(1 < Z < a) = 0.12$ $0.8413 + 0.12$
 $P(Z < 1) = 0.8413 \therefore P(Z < a) = 0.9613$
 $\therefore a = 1.77$
 b $P(a < Z < 1.6) = 0.787$
 $P(Z > 1.6) = 0.0548$
 $\therefore P(Z < a) = 1 - (0.787 + 0.0548)$
 $= 0.1582$
 $\therefore a = -1.00$



Exercise 15M

- 1 $X \sim N(30, \sigma^2)$ $P(X > 40) = 0.115$
 $Z = \frac{40 - 30}{\sigma} = \frac{10}{\sigma}$
 $\therefore \frac{10}{\sigma} = 1.2004 \therefore \sigma = 8.33$
 2 $X \sim N(\mu, 4^2)$ $P(X < 20.5) = 0.9$
 $Z = \frac{20.5 - \mu}{4} \therefore P\left(Z < \frac{20.5 - \mu}{4}\right) = 0.9$
 $\therefore \frac{20.5 - \mu}{4} = 1.28155$
 $\therefore \mu = 15.4$
 3 $X \sim N(\mu, \sigma^2)$ $P(X > 58.39) = 0.0217$
 $P(X < 41.82) = 0.0287$
 $Z = \frac{59.39 - \mu}{\sigma}$ $Z = \frac{41.82 - \mu}{\sigma}$
 $P\left(Z > \frac{58.39 - \mu}{\sigma}\right) = 0.0217$
 $\therefore P\left(Z < \frac{58.39 - \mu}{\sigma}\right) = 0.9783 \therefore \frac{58.39 - \mu}{\sigma} = 2.0198$
 $P\left(Z < \frac{41.82 - \mu}{\sigma}\right) = 0.0287 \therefore \frac{41.82 - \mu}{\sigma} = -1.9003$
 $\therefore \mu = 49.9 \quad \sigma = 4.23$

4 $X \sim N(\mu, \sigma^2)$ $P(X < 89) = 0.90$ $P(X < 94) = 0.95$

$$Z = \frac{89 - \mu}{\sigma} \quad Z = \frac{94 - \mu}{\sigma}$$

$$P\left(Z < \frac{89 - \mu}{\sigma}\right) = 0.90 \quad \therefore \frac{89 - \mu}{\sigma} = 1.28155$$

$$P\left(Z < \frac{94 - \mu}{\sigma}\right) = 0.95 \quad \therefore \frac{94 - \mu}{\sigma} = 1.64485$$

$$\therefore \mu = 71.4 \quad \sigma = 13.8$$

5 $X \sim N(136, \sigma^2)$ $P(X > 145) = 0.12$

$$Z = \frac{145 - 136}{\sigma} = \frac{9}{\sigma}$$

$$P\left(Z > \frac{9}{\sigma}\right) = 0.12 \quad \therefore P\left(Z < \frac{9}{\sigma}\right) = 0.88$$

$$\therefore \frac{9}{\sigma} = 1.175 \quad \therefore \sigma = 7.66 \text{ cm}$$

6 $X \sim N(\mu, 20^2)$ $P(X < 500) = 0.01$

$$Z = \frac{500 - \mu}{20}$$

$$P\left(Z < \frac{200 - \mu}{20}\right) = 0.01 \quad \therefore \frac{200 - \mu}{20} = -2.326$$

$$\therefore \mu = 546.5 \text{ or } 547 \text{ g}$$

7 $X \sim N(0.85, \sigma^2)$ $P(X < 1.1) = 0.74$

$$\text{a } Z = \frac{1.1 - 0.85}{\sigma} = \frac{0.25}{\sigma}$$

$$P\left(Z < \frac{0.25}{\sigma}\right) = 0.74 \quad \therefore \frac{0.25}{\sigma} = 0.6433$$

$$\therefore \sigma = 0.389 \text{ kg}$$

b $P(x > 1) = 0.350 = 35.0\%$

8 $X \sim N(\mu, 7^2)$ $P(X > 68) = 0.025$

$$Z = \frac{68 - \mu}{7}$$

$$P\left(Z > \frac{68 - \mu}{7}\right) = 0.025 \quad \therefore P\left(Z < \frac{68 - \mu}{7}\right) = 0.975$$

$$\therefore \frac{68 - \mu}{7} = 1.95996 \quad \therefore \mu = 54.3 \text{ cm}$$

9 $X \sim N(2.9, \sigma^2)$ $P(X > 3) = 0.35$

$$Z = \frac{3 - 2.9}{\sigma} = \frac{0.1}{\sigma}$$

$$P\left(Z > \frac{0.1}{\sigma}\right) = 0.35 \quad \therefore P\left(Z < \frac{0.1}{\sigma}\right) = 0.65$$

$$\therefore \frac{0.1}{\sigma} = 0.3853 \quad \therefore \sigma = 0.260 \text{ m}$$

10 a $X \sim N(\mu, \sigma^2)$ $P(X < 10.8) = 0.3$ $P(X > 154) = 0.2$

$$Z = \frac{10.8 - \mu}{\sigma} \quad Z = \frac{154 - \mu}{\sigma}$$

$$P\left(x = \frac{-7 \pm \sqrt{65}}{2}\right) = 0.3 \quad \therefore \frac{10.8 - \mu}{\sigma} = -0.5244$$

$$P\left(Z > \frac{154 - \mu}{\sigma}\right) = 0.2 \quad \therefore P\left(Z < \frac{154 - \mu}{\sigma}\right) = 0.8$$

$$\therefore \frac{154 - \mu}{\sigma} = 0.8416$$

$$\therefore \mu = 125.7 \quad \sigma = 33.67$$

$$\mu = 126 \quad \sigma = 33.7$$

b $P(X > 117) = 0.605 = 60.5\%$

yes, this is consistent with the normal distribution

11 $X \sim N(\mu, \sigma^2)$ $P(X > 495) = 0.95$

$$P(X > 490) = 0.99$$

$$\frac{495 - \mu}{\sigma} \quad \frac{490 - \mu}{\sigma}$$

$$P\left(Z > \frac{495 - \mu}{\sigma}\right) = 0.95 \quad \therefore P\left(Z < \frac{495 - \mu}{\sigma}\right) = 0.05$$

$$\therefore \frac{495 - \mu}{\sigma} = -1.64485 \quad (1)$$

$$P\left(Z > \frac{490 - \mu}{\sigma}\right) = 0.99 \quad \therefore P\left(Z < \frac{490 - \mu}{\sigma}\right) = 0.01$$

$$\therefore \frac{490 - \mu}{\sigma} = -2.32635 \quad (2)$$

Solving (1) and (2) simultaneously gives

$$\mu = 507.1, \sigma = 7.34$$



Review exercise

1 a $0.3 + \frac{1}{k} + \frac{2}{k} + 0.1 + 2.1 = 1$

$$\frac{3}{k} = 0.5 \quad \therefore k = 6$$

b $E(X) = (-2 \times 0.3) + (-1 \times \frac{1}{6}) + (1 \times 0.1)$

$$+ (2 \times 0.1)$$

$$= -\frac{7}{15}$$

2 a

x	1	2	3	4	5
P(X = x)	5c	8c	9c	8c	5c

$$35c = 1$$

$$\therefore c = \frac{1}{35}$$

b from symmetry, $E(X) = 3$

3 $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + x = 1 \quad \therefore x = \frac{3}{8}$

$$P(6) = P(2, 4) + P(3, 3) + P(4, 2)$$

$$= \frac{1}{4} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8} + \frac{3}{8} \times \frac{1}{4} = \frac{13}{64}$$

4 a

	2	2	4	4
1	2	2	4	4
2	4	4	8	8
3	6	6	12	12
4	8	8	16	16

possible values of P are 2, 4, 6, 8, 12, 16

b

x	2	4	6	8	12	16
P(X = x)	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

c $E(P) = \frac{2}{8} + \frac{8}{8} + \frac{6}{8} + \frac{16}{8} + \frac{12}{8} + \frac{6}{8}$
 $= 7.5$

d

x	£10	£5
P(X = x)	$\frac{1}{4}$	$\frac{3}{4}$

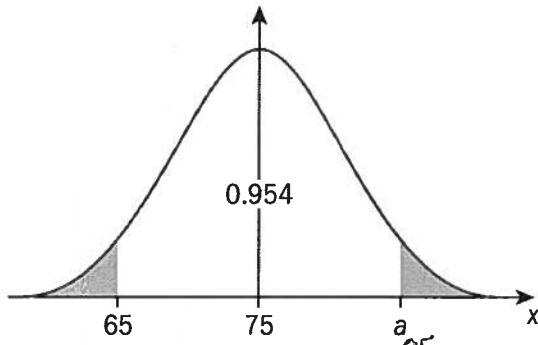
$E(X) = 10 \times \frac{1}{4} + 5 \times \frac{3}{4} = 6.25$

After 10 weeks, expected total = $6.25 \times 10 = £62.50$

5 $X \sim B(5, \frac{1}{3})$ $P(X=3) = \binom{5}{3} \times (\frac{1}{3})^3 \times (\frac{2}{3})^2$
 $= 10 \times \frac{1}{27} \times \frac{4}{9} = \frac{40}{243}$ *binompdf(5, 1/3, 3)*

6 $X \sim B(2, 0.1)$ $E(X) = nP = 2 \times 0.1 = 0.2$

7 a



$P(X < 65) = P(X > a)$
 From symmetry, $a = 85$

b $P(X > a) = \frac{1-0.954}{2} = 0.023$

4 a $X \sim B(10, 0.2)$

i $P(X=4) = 0.0881$

ii $P(X > 5) = 0.00637$

b $P(X=0) = 0.107$ $P(X=1) = 0.268$
 $P(X=2) = 0.302$

$P(X=3) = 0.201$ the probabilities continue to decrease after this

\therefore most likely number is 2

c $X \sim B(n, 0.2)$ $P(X \geq 1) > 0.95$

$1 - P(X=0) > 0.95$

$0.05 > P(X=0)$

$P(X=0) < 0.05$

$(0.8)^n < 0.05$

$n \log 0.8 < \log 0.05$

$n > \frac{\log 0.05}{\log 0.8}$

$n > 13.4$



\therefore need 14 points in this sample

5 $P(|Z| \leq a) = 0.85$

$P(-a \leq Z \leq a) = 0.85$

$\therefore P(Z \leq a) = 0.925$ $\therefore a = 1.44$

6 a $X \sim N(71, \sigma^2)$ $p(x < 80) = 0.85$

$Z = \frac{80-71}{\sigma} = \frac{9}{\sigma}$

$P(Z < \frac{9}{\sigma}) = 0.85$ $\therefore \frac{9}{\sigma} = 1.0364$ $\therefore \sigma = 8.68$

b $P(X > 65) = 0.755$

7 $X \sim N(\mu, \sigma^2)$ $P(X < 30) = 0.15$ $P(X > 50) = 0.1$

$Z = \frac{30-\mu}{\sigma}$ $Z = \frac{50-\mu}{\sigma}$

$P(Z < \frac{30-\mu}{\sigma}) = 0.15$ $\therefore \frac{30-\mu}{\sigma} = -1.03643$

$P(Z > \frac{50-\mu}{\sigma}) = 0.1$ $\therefore P(Z \leq \frac{50-\mu}{\sigma}) = 0.9$

$\therefore \frac{50-\mu}{\sigma} = 1.28155$

$\mu = 38.9$ hours $\sigma = 8.63$ hours (8.62 if round previous ans to 2 places)

8 a $X \sim N(\mu, 2)$ $P(x > 35) = 0.2$

$Z = \frac{35-\mu}{2}$

$P(Z > \frac{35-\mu}{2}) = 0.2$ $\therefore P(Z < \frac{35-\mu}{2}) = 0.8$

$\therefore \frac{35-\mu}{2} = 0.8416$

$\therefore \mu = 33.3$

b $X \sim B(5, 0.2)$ $P(X=0) = 0.328$

c $P(X \geq 2) = 0.263$

binompdf(5, 0.2, 0)
1 - binomcdf(5, 0.2, 1)



Review exercise

1 a $X \sim B(3, \frac{1}{3})$ $P(X \geq 1) = \frac{19}{27}$ *P(1 or 2)*

b

x	-5	1
P(X = x)	$\frac{8}{27}$	$\frac{19}{27}$

1 - binompdf(3, 1/3, 0)
sum = 1

c i $E(X) = -5 \times \frac{8}{27} + 1 \times \frac{19}{27} = -\frac{7}{9}$ or $-\$0.78$

\therefore lose $\$0.78$ (or $\$ \frac{7}{9}$)

ii $\frac{7}{9} \times 9 = 7$ \therefore lose $\$7$

2 $X \sim B(8, 0.3)$

a $P(X=3) = 0.254$ *Binompdf(8, 0.3, 3)*

b $P(X \geq 3) = 0.448$ *1 - Binomcdf(8, 0.3, 2)*

3 $X \sim B(6, \frac{1}{6})$ $P(X=3) = 0.05358$ *P(6)*

$Y \sim B(5, 0.05358)$ $P(Y=2) = 0.0243$

Binompdf(5, 1/6, 2)