Ch. 14 Non Calc Review

1) ⓐ $f(x) = \cos(1-2x)$ (CR)

$f'(x) = -\sin(1-2x) \cdot (-2) = \boxed{2\sin(1-2x)}$

ⓑ $y = \sin^3 x = (\sin x)^3$ (CR)

$y' = 3(\sin x)^2 \cdot \cos x = \boxed{3\sin^2 x \cdot \cos x}$

ⓒ $s(t) = e^{\tan t}$ (CR)

$s'(t) = e^{\tan t} \cdot \frac{1}{\cos^2 x} = \boxed{\frac{e^{\tan t}}{\cos^2 x} \text{ or } e^{\tan t} \cdot \sec^2 x}$

ⓓ $f(x) = \sqrt{\sin(x^2)} = (\sin(x^2))^{1/2}$ (CR/CR)

$f'(x) = 1/2\,(\sin(x^2))^{-1/2} \cdot \cos(x^2) \cdot 2x$

$= \boxed{\frac{x \cdot \cos(x^2)}{(\sin(x^2))^{1/2}} \text{ or } \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}}$

ⓔ $f(x) = x^2 \cdot \cos x$ (PR)

$f'(x) = x^2 \cdot (-\sin x) + 2x \cdot \cos x = \boxed{-x^2 \sin x + 2x\cos x}$

ⓕ $y = \ln(\tan x)$ (CR)

$y' = \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x} = \frac{\cos x}{\tan x} \cdot \frac{1}{\cos^2 x} = \boxed{\frac{1}{\sin x \cdot \cos x}}$

ⓖ $f(x) = (\ln x)(\sin x)$ (PR)

$f'(x) = \frac{1}{x}\sin x + \ln x \cdot \cos x = \boxed{\frac{\sin x}{x} + (\ln x)\cos x}$

(h) $y = 2\sin x \cos x$ (PR)

$y' = 2\sin x \cdot (-\sin x) + 2\cos x \cdot \cos x$

$\boxed{y' = -2\sin^2 x + 2\cos^2 x}$

$= 2\cos^2 x - 2\sin^2 x$

(or) $= 2(\cos^2 x - \sin^2 x)$

$\boxed{y' = 2\cos 2x}$

2) (a) $\int (4x^3 - \sin x)\,dx = \frac{1}{4}(4x^4) - (-\cos x) + C$

$= \boxed{x^4 + \cos x + C}$

(b) $\int \cos(3x)\,dx$ $\quad u = 3x \quad du = 3dx \quad \frac{1}{3}du = dx$

$\int \cos u \cdot \frac{1}{3}du$

$1/3 \int \cos u\,du = 1/3 \cdot \sin u + C = \boxed{1/3 \sin(3x) + C}$

(c) $\int \sin(4x+1)\,dx$ $\quad u = 4x+1 \quad du = 4dx \quad 1/4\,du = dx$

$\int \sin u \; 1/4\,du$

$1/4 \int \sin u\,du = 1/4(-\cos u) + C = \boxed{-1/4 \cos(4x+1) + C}$

(d) $\int x \cdot \cos(2x^2)\,dx$ $\quad u = 2x^2 \quad du = 4x\,dx \quad 1/4\,du = x\,dx$

$\int \cos(2x^2)\,x\,dx$

$\int \cos u \cdot 1/4\,du$

$1/4 \int \cos u\,du = 1/4 \cdot \sin u + C = \boxed{1/4 \sin(2x^2) + C}$

(e) $\cancel{\int \frac{\sin(2t+1)}{(\cos(2t+1))^4}}$

(e) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} dt = \int \sin(2t+1) \cdot \frac{1}{(\cos(2t+1))^2} dt$

$u = \cos(2t+1) \qquad du = -2\sin(2t+1)dt$

$-\frac{1}{2}du = \sin(2t+1)dt$

$\int -\frac{1}{2}du \cdot \frac{1}{u^2} = -\frac{1}{2}\int u^{-2}du = -\frac{1}{2}\cdot(-1)u^{-1} + C$

$\frac{1}{2}u^{-1} + C = \frac{1}{2u} + C = \boxed{\frac{1}{2\cos(2t+1)} + C}$

(f) $\int \frac{\sin(\ln x)}{x} dx = \int \sin(\ln x) \cdot \frac{1}{x} dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$\int \sin u \cdot du = -\cos u + C = \boxed{-\cos(\ln x) + C}$

(g) $\int x e^{\sin x^2} \cos x^2 dx$

$u = \sin x^2$

$du = \cos x^2 \cdot 2x\, dx$

$\frac{1}{2}du = x \cos x^2 dx$

$\int e^{\sin x^2} \cdot x \cos x^2 dx$

$\int e^u \cdot \frac{1}{2}du = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}e^{\sin x^2} + C}$

(h) $\int \frac{6\cos x}{(2+\sin x)^2} dx = 6\int \cos x \cdot \frac{1}{(2+\sin x)^2} dx$

$u = 2 + \sin x$

$du = \cos x\, dx$

$6\int \frac{1}{u^2} du = 6\int u^{-2}du = 6(-1)u^{-1} + C = \frac{-6}{u} + C$

$\boxed{\frac{-6}{2+\sin x} + C}$

3) ⓐ $\int_{-\pi/3}^{\pi/3} \sin x\,dx = -\cos x \Big]_{-\pi/3}^{\pi/3}$

S A ← $\pi/3$
T C ← $-\pi/3$

$-\cos(\pi/3) - (-\cos(-\pi/3)) = -(1/2) + (1/2) = \boxed{0}$

ⓑ $\int_0^{\pi} (1+\sin x)\,dx = x + -\cos x \Big]_0^{\pi}$

(-1,0) (1,0)

$(\pi - \cos\pi) - (0 - \cos 0) = \pi - (-1) - (0-1)$
$= \pi + 1 + 1 = \boxed{\pi + 2}$

ⓒ $\int_0^{\pi} (\sin x + \cos(2x))\,dx$

$\int_0^{\pi} \sin x\,dx \quad + \quad \int_0^{\pi} \cos(2x)\,dx$

$u = 2x$
$du = 2dx$
$\frac{1}{2}du = dx$
if $x = \pi$ $u = 2\pi$
if $x = 0$ $u = 0$

$-\cos x \Big]_0^{\pi}$
$-\cos\pi - (-\cos 0)$
$-(-1) + (1)$
$1 + 1 = 2$

$\int_0^{2\pi} \cos u \cdot \frac{1}{2}du$
$\frac{1}{2} \int_0^{2\pi} \cos u\,du$
$\frac{1}{2} \cdot \sin u \Big]_0^{2\pi} = \frac{1}{2}\sin(2x) \Big]_0^{2\pi}$
$\frac{1}{2}(\sin(4\pi) - \sin 0) = \frac{1}{2}(0-0) = 0$

$2 + 0 = \boxed{2}$

ⓓ $\int_0^{\pi/2} 5\sin^{3/2}x\cos x\,dx = \int_0^{\pi/2} 5(\sin x)^{3/2}\cos x\,dx$

$u = \sin x$
$du = \cos x\,dx$
$5du = 5\cos x\,dx$
if $x = \pi/2$ $u = 1$
if $x = 0$ $u = 0$

$\int_0^{\pi/2} (\sin x)^{3/2} 5\cos x\,dx = \int_0^{1} u^{3/2} \cdot 5du$

$5\int_0^1 u^{3/2}du = 5 \cdot \frac{2}{5} u^{5/2} \Big]_0^1 = 2u^{5/2} \Big]_0^1$

$2(1^{5/2} - 0^{5/2}) = 2((\sqrt{1})^5 - (\sqrt{0})^5) = 2(1-0) = \boxed{2}$

4) $y = \cos(3x-6)$ pt $(2,1)$

$y' = -\sin(3x-6)\cdot 3$

$y' = -3\sin(3x-6)$

$m = -3\sin(3\cdot 2-6) = -3\sin 0 = -3(0) = 0$

$m=0$ $\perp m =$ undefined ∴ so vertical line

normal line goes through $(2,1)$

so $\boxed{x=2}$

5) $y = \sin(\frac{1}{2}x)$ $0 \le x \le \pi$ parallel to $y = \frac{1}{4}x+3$

$y' = \frac{1}{2}\cos(\frac{1}{2}x)$

$m = \frac{1}{4}$ if parallel

$\frac{1}{4} = \frac{1}{2}\cos(\frac{1}{2}x)$

$\frac{1}{2} = \cos(\frac{1}{2}x)$ Let $A = \frac{1}{2}x$

$\cos A = \frac{1}{2}$

Ref $\pi/3$ S A T C

$A = \pi/3$ or $5\pi/3$

$\frac{1}{2}x = \pi/3$ or $5\pi/3$

$x = 2\pi/3$ or $\cancel{10\pi/3}$

must be $0 \le x \le \pi$

if $x = 2\pi/3$

$y = \sin(\frac{1}{2}\cdot\frac{2\pi}{3})$

$= \sin(\frac{\pi}{3}) = \sqrt{3}/2$

$\boxed{\text{pt } (2\pi/3, \sqrt{3}/2)}$

6) $\int f'(x) = f(x) + C$ pt on $f(x)$ $(0,2)$

$\int (x-\sin x)dx = \frac{1}{2}x^2 - (-\cos x) + C = \frac{1}{2}x^2 + \cos x + C$

$2 = \frac{1}{2}(0)^2 + \cos(0) + C$

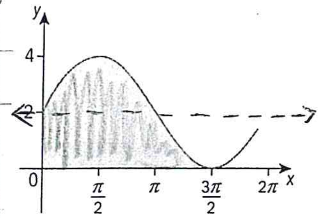
$2 = 0 + 1 + C$

$2 = 1 + C$

$C = 1$

$\boxed{y = \frac{1}{2}x^2 + \cos x + 1}$

7) $f(x) = p\sin(x) + q,\ p, q \in \mathbb{N}$



ⓐ vertical shift $= 2 = q$

amplitude $= 2 = p$

ⓑ $\int_0^{3\pi/2} (2\sin x + 2)\,dx$

$-2\cos x + 2x \Big]_0^{3\pi/2}$

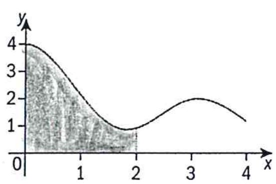
$-2\left(\cos 3\pi/2 - \cos 0\right) + 2\left(3\pi/2 - 0\right)$

$-2(0 - 1) + 2(3\pi/2)$

$\boxed{2 + 3\pi}$

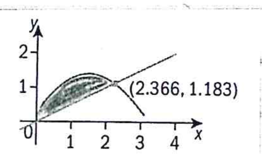
ch. 14 Calc Review

1) ⓐ



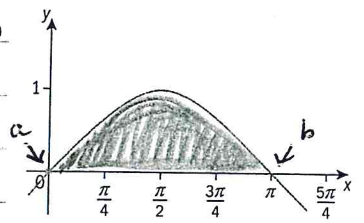
$\int_0^2 (2\cos^2 x + \cos x + 1)\,dx \approx \boxed{4.53}$

ⓑ

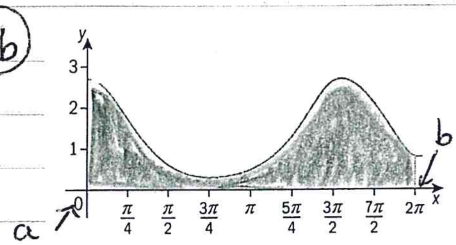


$\int_0^{2.366} (\sqrt{2\sin x} - 0.5x)\,dx \approx \boxed{1.36}$

2) ⓐ



ⓑ



$\int_a^b \pi (\sin x)^2\,dx$

$\int_0^{\pi} \pi (\sin x)^2\,dx = \boxed{4.93}$

$\int_a^b \pi \left(e^{\cos x}\right)^2 dx$

$\int_0^{2\pi} \pi \left(e^{\cos x}\right)^2 dx = \boxed{45.0}$

3) $\int_0^k \cos x = 0.942$ $\quad 0 < k < \pi/2$

$\sin x \Big]_0^k = 0.942$ $\quad \sin k - \sin 0 = 0.942$

$\sin k - 0 = 0.942$

$\sin k = 0.942$

$\sin^{-1}(0.942) = k$ $\quad$ S A / T C

$\boxed{k = 1.23}$

4) $s(t) = 2e^{\cos(5t)} - 4$

@ (i) $s'(t) = 2e^{\cos(5t)} \cdot (-\sin(5t)) \cdot 5$

$s'(t) = -10\sin(5t)e^{\cos(5t)}$

(ii) $s'(t) = -10\sin(5t)e^{\cos(5t)}$

$s''(t) = -10(\sin(5t)e^{\cos(5t)} \cdot -\sin(5t)\cdot 5 +$
$\cos(5t)\cdot 5 \cdot e^{\cos(5t)})$

$50\sin^2(5t)e^{\cos(5t)} - 50\cos(5t)e^{\cos(5t)}$

$\boxed{50e^{\cos(5t)}(\sin^2(5t) - \cos(5t)) = s''(t)}$

(iii) 2nd derivative test ... if min at $t = \pi/5$
$s'(\pi/5) = 0$ and $s''(\pi/5) > 0$

$s'(\pi/5) = -10\sin(5\cdot\pi/5)e^{\cos(5\cdot\pi/5)}$
$= -10\sin\pi\, e^{\cos\pi} = -10(0)e^{-1} = 0$ ✓

$s''(\pi/5) = 50e^{\cos(5\cdot\pi/5)}((\sin(5\cdot\pi/5))^2 - \cos(5\cdot\pi/5))$
$50e^{\cos\pi}((\sin\pi)^2 - \cos\pi)$
$50e^{-1}((0)^2 - (-1))$
$50/e\,(1) = 50/e > 0$ ✓

4 (b) total distance $= \int |v(t)|$

$s'(t) = v(t) = -10\sin(5t)e^{\cos(5t)}$

$$\int_0^2 \left|-10\sin(5t)e^{\cos(5t)}\right| dt = \boxed{14.2\text{ m}}$$