Non-Calc Solutions

1) $\vec{AB} = \begin{pmatrix}-2\\3\\5\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}-3\\1\\2\end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix}7\\0\\-1\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}6\\-2\\-4\end{pmatrix}$

$\vec{AB}$ and $\vec{AC}$ share a point and $2\vec{AB} = \vec{AC}$ so parallel
so collinear

2) Show two vectors are perpendicular

$\vec{AB} = \begin{pmatrix}2\\2\\0\end{pmatrix} - \begin{pmatrix}5\\-1\\6\end{pmatrix} = \begin{pmatrix}-3\\3\\-6\end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix}-3\\-5\\8\end{pmatrix} - \begin{pmatrix}5\\-1\\6\end{pmatrix} = \begin{pmatrix}-8\\-4\\2\end{pmatrix}$

$-3(-8) + 3(-4) + -6(2) = 24 + -12 + -12 = 0$

Since $\vec{AB} \perp \vec{AC}$ They form a rt angle so rt triangle.

3) $a+b = \begin{pmatrix}6\\2\\-8\end{pmatrix}$ $\quad a-b = \begin{pmatrix}4\\-4\\2\end{pmatrix}$

$6(4) + 2(-4) + -8(2)$

$24 + -8 + -16 = 0$

so perpendicular

4) $r_1 = r_2$ $\quad \begin{pmatrix}0\\6\\-1\end{pmatrix} + s\begin{pmatrix}7\\3\\1\end{pmatrix} = \begin{pmatrix}3\\1\\2\end{pmatrix} + t\begin{pmatrix}2\\4\\-1\end{pmatrix}$

$7s = 3 + 2t$

$6 + 3s = 1 + 4t$

$-1 + 1s = 2 - t \rightarrow s = 3 - t$

substitution

$7(3-t) = 3 + 2t$

$21 - 7t = 3 + 2t$

$t = 2$

$s = 1$

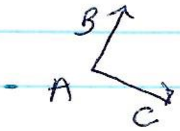
plug in either to find P

$\begin{pmatrix}0\\6\\-1\end{pmatrix} + 1\begin{pmatrix}7\\3\\1\end{pmatrix} = \boxed{\begin{pmatrix}7\\9\\0\end{pmatrix}}$

5) (a) $\vec{AB} = \begin{pmatrix} 1 \\ 7 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$ $\quad \vec{AC} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

(b) $3(-1) + 3(-2) = -3 + -6 = -9$

(c) $\cos B\hat{A}C = \dfrac{\vec{AB} \cdot \vec{AC}}{|AB| \cdot |AC|} = \dfrac{-9}{\sqrt{3^2+3^2} \cdot \sqrt{(-1)^2+(-2)^2}} = \dfrac{-9}{\sqrt{18} \cdot \sqrt{5}}$



$\dfrac{-9}{\sqrt{90}} = \dfrac{-9}{3\sqrt{10}} = \dfrac{-3}{\sqrt{10}} = \dfrac{-3}{\sqrt{2}\sqrt{5}}$ ✓

6) (a) $\begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix} + 4\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} -2 \\ 10 \\ 1 \end{pmatrix}}$

(b) $\begin{pmatrix} -2 \\ 10 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -12 \\ 7 \end{pmatrix} + t\begin{pmatrix} -1 \\ 11 \\ -3 \end{pmatrix}$

$-2 = -t \longrightarrow t = 2$
$10 = -12 + 11t \longrightarrow t = 2$
$1 = 7 - 3t \longrightarrow t = 2$

Since all t's are the same, $(-2, 10, 1)$ is on the line.

7) (a) use $B(2,2,4)$ as "a" and "b" is a multiple of $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

$r = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix} + s\begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$ (for Example)

(b) show that $x = -1$
multiply the directional vectors (b)

$\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ 1 \end{pmatrix} = 0 \qquad 1(1) + 3(x) + 2(1) = 0$

$1 + 3x + 2 = 0$

$3x = -3$

$\boxed{x = -1}$

8) ⓐ $\begin{pmatrix}-4\\3\end{pmatrix} + t\begin{pmatrix}4\\17\end{pmatrix} = \begin{pmatrix}4\\9\end{pmatrix} + s\begin{pmatrix}-12\\5\end{pmatrix}$

$-4 + 4t = 4 - 12s \rightarrow t = 2 - 3s$

$3 + 17t = 9 + 5s \leftarrow$ substitution

$$3 + 17(2-3s) = 9 + 5s$$
$$3 + 34 - 51s = 9 + 5s$$
$$28 = 56s$$
$$s = 1/2$$

Plug in s to find both t's

$-4 + 4t = 4 - 12(1/2) \rightarrow t = 0.5$

$3 + 17t = 9 + 5(1/2) \rightarrow t = 0.5$

Since all t's are the same it collides at $t = 1/2$ hr. or 12:30 pm

position: $\begin{pmatrix}-4\\3\end{pmatrix} + (1/2)\begin{pmatrix}4\\17\end{pmatrix} = \begin{pmatrix}-2\\11.5\end{pmatrix}$

ⓑ at 12:15 $t = 1/4$ hr. $\begin{pmatrix}-4\\3\end{pmatrix} + (1/4)\begin{pmatrix}4\\17\end{pmatrix} = \begin{pmatrix}-3\\7.25\end{pmatrix}$

new vector for ship A

$\begin{pmatrix}-3\\7.25\end{pmatrix} + t\begin{pmatrix}16\\17\end{pmatrix}$ where $t =$ time since 12:15

position at 12:30

Ship A: $t = 1/4$
use new equation

$\begin{pmatrix}-3\\7.25\end{pmatrix} + 1/4\begin{pmatrix}16\\17\end{pmatrix} = \begin{pmatrix}1\\11.5\end{pmatrix}$

ship B: $t = 1/2$
use B. equation

$\begin{pmatrix}4\\9\end{pmatrix} + (1/2)\begin{pmatrix}-12\\5\end{pmatrix} = \begin{pmatrix}-2\\11.5\end{pmatrix}$

distance: $\begin{pmatrix}1\\11.5\end{pmatrix} - \begin{pmatrix}-2\\11.5\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}$ $\sqrt{3^2+0^2} = \boxed{3 \text{ km}}$