

# Calculator Review

$$\textcircled{1} \cos \theta = \frac{a \cdot b}{|a| \cdot |b|} = \frac{3(2) + 5(-4)}{(\sqrt{3^2 + 5^2})(\sqrt{2^2 + (-4)^2})} = \frac{-14}{\sqrt{34}\sqrt{20}} = \frac{-14}{\sqrt{680}}$$

$$\cos \theta = -14/\sqrt{680} \quad \boxed{\theta = 122^\circ}$$

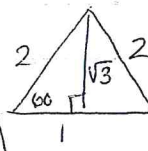
$$\textcircled{2} \textcircled{a} \vec{QR} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix} \quad \vec{QP} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\textcircled{b} \cos \hat{PQR} = \frac{QR \cdot QP}{|QR| \cdot |QP|} = \frac{(-1)(0) + (0)(-1) + (5)(1)}{(\sqrt{(-1)^2 + 0^2 + 5^2})(\sqrt{0^2 + (-1)^2 + (1)^2})}$$

$$\cos \hat{PQR} = \frac{5}{\sqrt{26}\sqrt{2}} \quad \cos \hat{PQR} = \frac{5}{\sqrt{52}} \quad \boxed{\hat{PQR} = 46.1^\circ}$$

$$\textcircled{c} \frac{1}{2} \sqrt{26}\sqrt{2} \sin 46.1^\circ = \boxed{2.60}$$

$$\textcircled{3} \textcircled{a} \textcircled{i} 4j \quad \textcircled{ii} i + \sqrt{3}k \quad \textcircled{iii} 2i + 4j$$



$$\textcircled{b} \vec{BC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ \sqrt{3} \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -\sqrt{3} \end{pmatrix} = \boxed{-i + 4j - \sqrt{3}k}$$

$$\vec{BD} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ \sqrt{3} \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -\sqrt{3} \end{pmatrix} = \boxed{i + 4j - \sqrt{3}k}$$

$$\textcircled{c} \textcircled{i} |\vec{BC}| = \sqrt{1 + 16 + 3} = \sqrt{20} = \boxed{2\sqrt{5}}$$

$$\textcircled{ii} |\vec{BD}| = \sqrt{1 + 16 + 3} = \sqrt{20} = \boxed{2\sqrt{5}}$$

$$\textcircled{iii} (-1)(1) + (4)(4) + (-\sqrt{3})(-\sqrt{3}) = -1 + 16 + 3 = \boxed{18}$$

$$\textcircled{d} \cos \theta = \frac{18}{\sqrt{20}\sqrt{20}} \quad \cos \theta = \frac{18}{20} \quad \boxed{\theta = 25.8^\circ}$$

4) a)  $\begin{pmatrix} x \\ x-2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x^2 \\ -2x \\ -12x \end{pmatrix} = 0$   $x^3 + -2x(x-2) + -12x = 0$   
 $x^3 - 2x^2 + 4x - 12x = 0$   
 $x^3 - 2x^2 - 8x = 0$   
 $x(x^2 - 2x - 8) = 0$   
 $x(x-4)(x+2) = 0$   
 $x = 0, 4, -2$

b)  $\begin{pmatrix} -1 \\ -1-2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} (-1)^2 \\ -2(-1) \\ -12(-1) \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 12 \end{pmatrix}$

$$\cos \theta = \frac{a \cdot b}{|a||b|} = \frac{(-1)(1) + (-3)(2) + (1)(12)}{\sqrt{(-1)^2 + (-3)^2 + (1)^2} \sqrt{(1)^2 + 2^2 + 12^2}}$$

$$\cos \theta = \frac{5}{\sqrt{11} \sqrt{149}} \quad \theta = 82.9^\circ$$

5) a)  $\vec{OP} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$   $\vec{PQ} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 6 \\ 2 \end{pmatrix} = (1)(0) + (-1)(6) + (3)(2) = 0 + -6 + 6 = 0 \checkmark$$

b)  $r = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 6 \\ 2 \end{pmatrix}$  (one possible answer)

c)  $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 6 \\ 2 \end{pmatrix}$

$$\begin{aligned} 2 + 1\mu &= 1 & \mu &= -1 \\ -1 + -3\mu &= -1 + 6\lambda & \leftarrow & -1 + -3(-1) = -1 + 6\lambda \\ 2 + -2\mu &= 3 + 2\lambda & & \lambda = \frac{1}{2} \end{aligned}$$

plug back in  
 $\mu$  or  $\lambda$   
 $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

(d) Use direction vectors

$$\begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \\ 2 \end{pmatrix} \quad a \cdot b = 0 + -18 + -4 = -22$$

$$|a| = \sqrt{1+9+4} = \sqrt{14} \quad |b| = \sqrt{0+36+4} = \sqrt{40}$$

$$\cos \theta = \frac{-22}{\sqrt{14}\sqrt{40}} \Rightarrow \boxed{\theta = 158^\circ}$$

(b) a  $\begin{pmatrix} 6 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} = \boxed{\begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix}}$

b "b" = direction vector =  $\frac{\text{displacement}}{\text{time}} = \frac{1}{2} \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$

s "a" = initial position =  $\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$

$$\boxed{r = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}}$$

c plug in  $t=0$   $\boxed{\begin{pmatrix} 36 \\ 18 \\ 0 \end{pmatrix}}$

d  $|b| = \sqrt{(-3)^2 + (-4)^2 + (1)^2} = \sqrt{9+16+1} = \sqrt{26} = \cancel{5.1}$

$$\boxed{5.1 \text{ mps}}$$

e  $\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 36 \\ 18 \\ 0 \end{pmatrix} + s \begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix}$

$$3t = 36 - 3s \quad 3t = 36 - 18$$

$$-t = 18 - 4s$$

$$\boxed{6 = s}$$

$$\boxed{t = 6}$$

$$\boxed{6 \text{ seconds}}$$

f plug 6 into one equation

$$\begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + 6 \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + \begin{pmatrix} 18 \\ -6 \\ 0 \end{pmatrix} = \boxed{\begin{pmatrix} 18 \\ -6 \\ 6 \end{pmatrix}}$$