IB Math Studies Ch 13 Calculus Review

REVIEW SET 20A

1 The total number of televisions sold over many months is shown on the graph alongside.

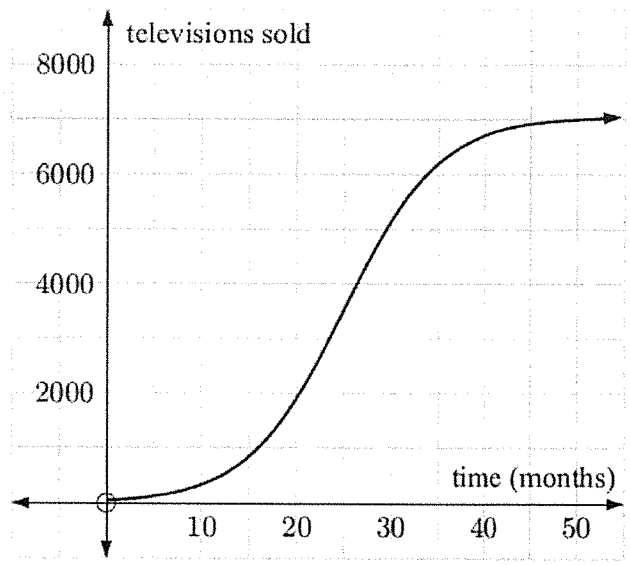
Estimate the rate of sales:

~~**a**~~ from 40 to 50 months

b from 0 to 50 months

~~**c**~~ at 20 months.

PRINTABLE GRAPH



Part d Find the gradient of the line segment from $t = 20$ months to $t = 50$ months.

2 Use the rules of differentiation to find $f'(x)$ for $f(x)$ equal to:

a $7x^3$ **b** $3x^2 - x^3$ **c** $(2x-3)^2$ **d** $\dfrac{7x^3 + 2x^4}{x^2}$

3 Consider $f(x) = x^4 - 3x - 1$. Find: **a** $f'(x)$ **b** $f'(2)$ **c** $f'(0)$.

4 Find the equation of the tangent to $y = -2x^2$ at the point where $x = -1$.

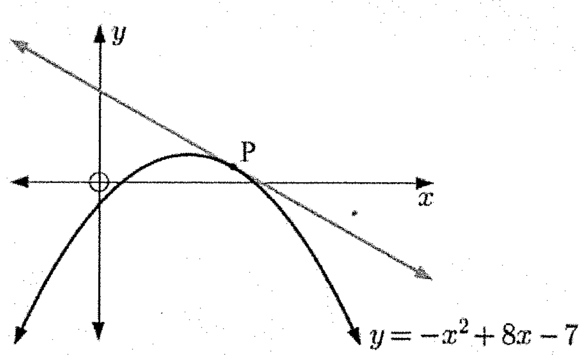
5 Consider the function $f(x) = -2x^2 + 5x + 3$. Find:

a the average rate of change from $x = 2$ to $x = 4$

~~**b**~~ the instantaneous rate of change at $x = 2$.

6 Find the equation of the normal to $y = x^3 + 1x - 2$ at the point where $x = 1$

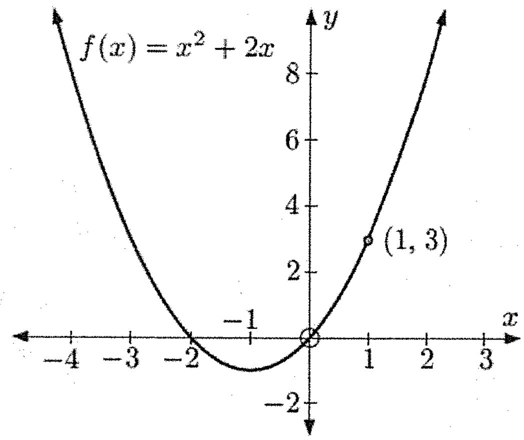
7 The tangent shown has gradient -4. Find the coordinates of P.



8 The tangent to $y = ax^3 - 3x + 3$ at the point where $x = 2$, has a gradient of 21. Find a.

REVIEW SET 20B

1 Consider the function $f(x) = x^2 + 2x$, which has the graph shown.



a Find the gradient of the line which passes through $(1,\ 3)$ and the point on $f(x)$ with x-coordinate:

~~i~~ 2 **ii** 1.5 ~~iii~~ 1.1

b Find $f'(x)$.

c Find the gradient of the tangent to $f(x)$ at $(1,\ 3)$. Compare this with your answers to **a**.

2 Find $\dfrac{dy}{dx}$ for:

a $y = 3x^2 - x^4$ **b** $y = \dfrac{x^3 - x}{x^2}$ **c** $y = 2x + x^{-1} - 3x^{-2}$

~~3~~ Find the equation of the tangent to $y = x^3 - 3x + 5$ at the point where $x = 2$.

4 Find all points on the curve $y = 2x + 2x^{-1}$ where the tangent is horizontal.

~~5~~ If $f(x) = 7 + x - 3x^2$, find: **a** $f(3)$ **b** $f'(3)$.

not on our test **6** Find the coordinates of the point where the normal to $y = x^2 - 7x - 44$ at $x = -3$ meets the curve again.

~~7~~ The tangent to $f(x) = a - \dfrac{b}{x^2}$ at $(-1,\ -1)$ has equation $y = -6x - 7$. Find the values of a and b.

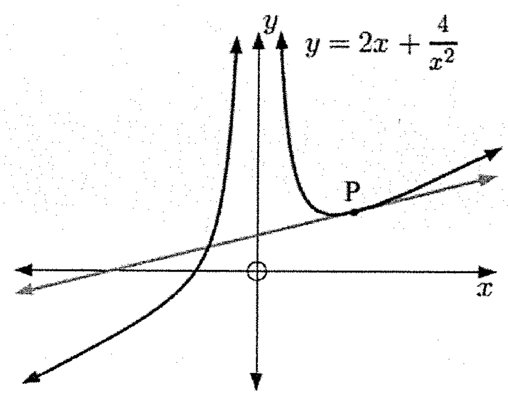
8 The tangent shown has a gradient of 1.

a Find the coordinates of P.

b Find the equation of the tangent.

c Find where the tangent cuts the x-axis.

d Find the equation of the normal at P.



REVIEW SET 20C

1 Use the rules of differentiation to find $f'(x)$ for $f(x)$ equal to:

a $x^4 + 2x^3 + 3x^2 - 5$ **b** $2x^{-3} + x^{-4}$ **c** $\dfrac{1}{x} - \dfrac{4}{x^2}$

2 Find the gradient of $f(x)$ at the given point for the following functions:

a $f(x) = x^2 - 3x$ at $x = -1$ **b** $f(x) = -3x^2 + 4$ at $x = 2$

c $f(x) = x + \dfrac{2}{x}$ at $x = 3$ **d** $f(x) = x^3 - x^2 - x - 2$ at $x = 0$

3 Find the equation of the tangent to $y = \dfrac{12}{x^2}$ at the point $(1, 12)$.

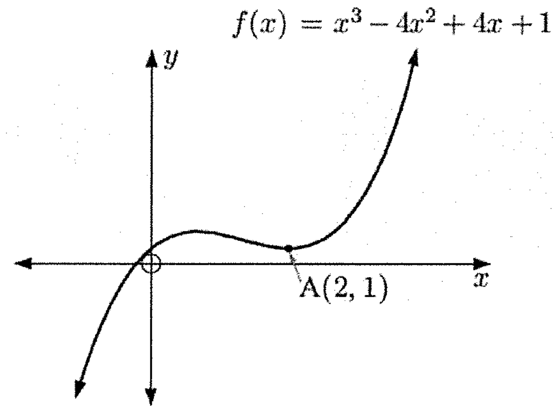
4 Sand is poured into a bucket for 30 seconds. After t seconds, the weight of sand is $S(t) = 0.3t^3 - 18t^2 + 550t$ grams.
Find and interpret $S'(t)$.

5 Find the equation of the normal to $y = 2 + \dfrac{1}{x} + 3x$ at the point where $x = 1$.

6 The tangent to $y = x^3 - 2x^2 + ax - b$ at $(2, -1)$ has equation $y = 7x - 15$.
Find the values of a and b.

7 Find the coordinates of the point where the normal to $y = -3x^3 + 5x - 1$ at $x = 0$ meets the curve again.

8 The graph of $f(x) = x^3 - 4x^2 + 4x + 1$ is shown alongside.



a Find $f'(x)$.

b Find and interpret $f'(1)$.

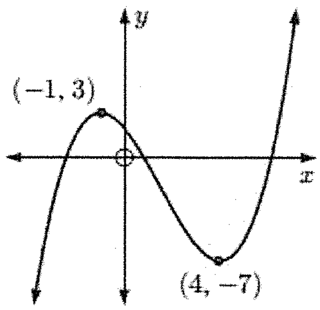
c The graph has a minimum turning point at A(2, 1).

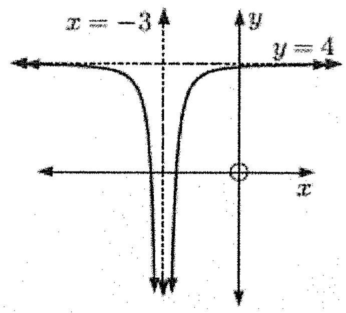
i Find the gradient of the tangent at A.

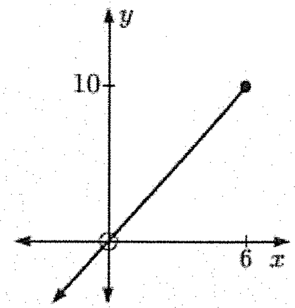
ii Find the equation of the tangent at A.

REVIEW SET 21A

1 Find intervals where the graphed function is increasing or decreasing.

a


b


c


2 Consider the function $f(x) = x^3 - 3x$.

a Determine the y-intercept of the function.

b Find $f'(x)$.

c Hence find the position and nature of any stationary points.

d Sketch the graph of the function, showing the features you have found.

3 Consider the function $f(x) = 3x + 2 + \dfrac{48}{x}$.

a Find $f'(x)$ and draw its sign diagram.

b Find and classify all stationary points of the function.

c Sketch the graph of $y = f(x)$.

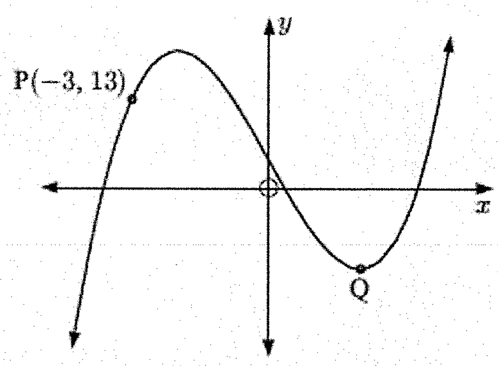
4 An open box is made by cutting squares out of the corners of a 24 cm by 24 cm square sheet of tinplate. Use calculus techniques to determine the size of the squares that should be removed to maximise the volume of the box.

5 The graph of $f(x) = x^3 - 12x + 4$ is shown.

a Find $f'(x)$.

b Find the gradient of the tangent to the graph at P.

c The graph has a local minimum at Q.
Find the coordinates of Q.



6 A factory makes x thousand chopsticks per day with a cost of $C(x) = 0.4x^2 + 1.6x + 150$ dollars. Packs of 1000 chopsticks sell for $28.

a Using calculus, find the production level that maximises daily profit.

b Determine the profit at that production level.

REVIEW SET 21B

1 Find the absolute maximum and absolute minimum values of $x^3 - 3x^2 + 5$ for $-1 \leqslant x \leqslant 4$.

2 Consider the function $f(x) = 2x^3 - 3x^2 - 36x + 7$.

 a Find $f'(x)$.

 b Find and classify all stationary points.

 c Find intervals where the function is increasing or decreasing.

 d Sketch the graph of $y = f(x)$, showing all important features.

3 An astronaut standing on the moon throws a ball into the air. The ball's height above the surface of the moon is given by $H(t) = 1.5 + 19t - 0.8t^2$ metres, where t is the time in seconds.

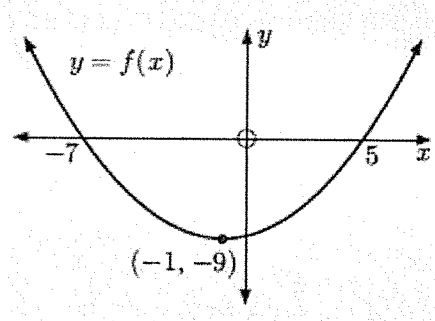
 a Find $H'(t)$ and state its units.

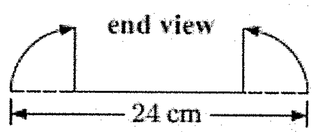
 b Calculate $H'(0)$, $H'(10)$, and $H'(20)$. Interpret these values, including their sign.

 c How long is the ball in the 'air' for?

4 For the function $y = f(x)$ shown, draw a sign diagram of:

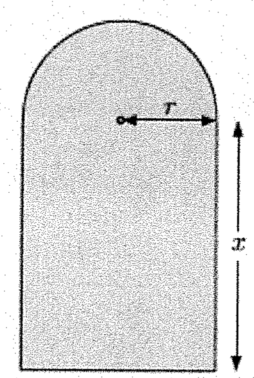
 a $f(x)$ b $f'(x)$.



5 

A rectangular gutter is formed by bending a 24 cm wide sheet of metal as illustrated. Where must the bends be made to maximise the capacity of the gutter?

6 The new college lawn will be a rectangle with a semi-circle on one of its sides. Its perimeter will be 200 m.



 a Find an expression for the perimeter of the lawn in terms of r and x.

 b Find x in terms of r.

 c Show that the area of the lawn A can be written as $A = 200r - r^2\left(2 + \frac{\pi}{2}\right)$.

 d Use calculus to find the values of x and r which maximise the area of the lawn.

7 At time t years after mining begins on a mountain of iron ore, the rate of mining is given by $R(t) = \dfrac{1000 \times 3^{0.03t}}{25 + 2^{0.25t-10}}$ million tonnes per year, $t \geqslant 0$.

 a Graph $R(t)$ against t for $0 \leqslant t \leqslant 100$.

 b At what rate will the ore be mined after $t = 20$ years?

 c What will be the maximum rate of mining, and at what time will it occur?

 d When will the rate of mining be 100 million tonnes per year?

REVIEW SET 21C

3 The cost per hour of running a barge up the Rhein is given by $C(v) = 10v + \frac{90}{v}$ euros, where v is the average speed of the barge.

a Find the cost of running the barge for:

i two hours at 15 km h^{-1} **ii** 5 hours at 24 km h^{-1}.

b Find the rate of change in the cost of running the barge at speeds of:

i 10 km h^{-1} **ii** 6 km h^{-1}.

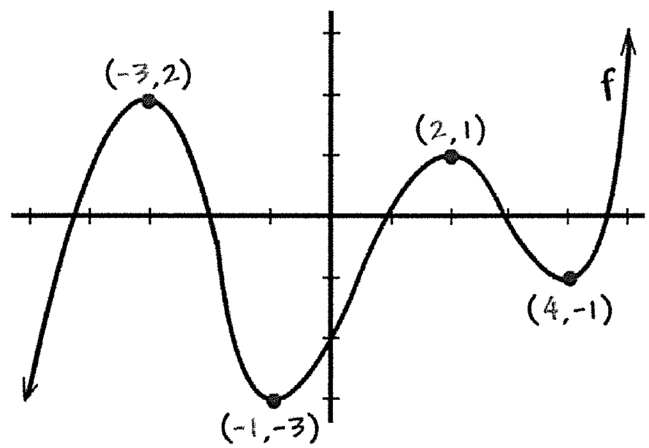
c At what speed will the cost per hour be a minimum?

8 Given $f(x) = 3x^2 + 2x + 5$

a. Find $f'(x)$ at $x = 2$

b. Is $f(x)$ increasing or decreasing at $x = 2$

9 Answer the questions below about the given graph of $f(x)$:



a. Where is the graph increasing?

b. Where is the graph decreasing?

c. Where is the gradient positive?

d. Where is the gradient negative?

e. Where is the gradient zero?

f. Where is $f'(x)$ positive?

g. Where is $f'(x)$ negative?

h. Where is $f'(x) = 0$?

i. Where is $f'(x) > 0$?

j. Where is $f'(x) < 0$?

k. What are the local maximum values?

Absolute maximum values?

l. What are the local minimum values?
Absolute minimum values?