Review Solutions Ch. 13 Review

Review Set 20A

1) b) $(0,0)\ (50,7000) = \frac{7000-0}{50-0} = 7000/50 = 140$ tvs per month

d) $(20,2000)(50,7000) = \frac{7000-2000}{50-20} = 5000/30 = 500/3$

2) differentiation = find derivative

a) $21x^2 = f'(x)$

b) $6x - 3x^2 = f'(x)$

c) $(2x-3)^2 = (2x-3)(2x-3) = 4x^2 - 12x + 9 \Rightarrow 8x - 12 = f'(x)$

d) $7x^3+2x^4 / x^2 = 7x + 2x^2 \Rightarrow 7 + 4x = f'(x)$

3) a) $f'(x) = 4x^3 - 3$

b) $f'(2) = 4(2)^3 - 3 = 4(8) - 3 = 32 - 3 = \boxed{29}$

c) $f'(0) = 4(0)^3 - 3 = 0 - 3 = \boxed{-3}$

4) find y first...

$y = -2(-1)^2 = -2(1) = -2$

$\boxed{\text{pt } (-1, -2)}$

find derivative at $x = -1$

$y' = -4x$

$= -4(-1) = \boxed{4 = m}$

$y - y_1 = m(x - x_1)$

$y + 2 = 4(x + 1)$

$y + 2 = 4x + 4$

$\boxed{y = 4x + 2}$

5) $f(2) = -2(2)^2 + 5(2) + 3 = 5$

$f(4) = -2(4)^2 + 5(4) + 3 = -9$

$(2,5)\ (4,-9)$

$\frac{-9-5}{4-2} = \frac{-14}{2} = \boxed{-7}$

6) find y first...

$y = (1)^3 + (1) - 2 = 0$

$\boxed{\text{pt } (1, 0)}$

find derivative at $x = 1$

$y' = 3x^2 + 1$

$= 3(1)^2 + 1 = 4$ $\boxed{m = -1/4}$

$y - 0 = -1/4(x - 1)$

$\boxed{y = -1/4x + 1/4}$

Review Set 20A continued

7) $m=-4$

$y' = -2x+8$ → point is tangent at $x=6$ → $y=-(6)^2+8(6)-7$

$-4 = -2x+8$ → $y=5$

$-12 = -2x$

$x=6$ → $\boxed{(6,5)}$

8) $m=21$ at $x=2$

$y' = 3ax^2-3$

$21 = 3a(2)^2-3$

$21 = 12a-3$

$24 = 12a$ → $\boxed{a=2}$

Review Set 20B

1) a) $x=1.1$

$y=(1.1)^2+2(1.1) = 3.41$

$(1,3)\ (1.1, 3.41)$ → $\frac{3.41-3}{1.1-1} = \frac{0.41}{.1} = \boxed{4.1}$

b) $f'(x) = 2x+2$

c) $x=1$ $f'(1) = 2(1)+2 = 4$

Since the distance between $(1,3)$ and $(1.1, 3.41)$ is small, Their slopes are close to The same.

2) dy/dx means y'

a) $y' = 6x-4x^3$

b) $\frac{x^3-x}{x^2} = x-\frac{1}{x} = x-x^{-1}$

& $y' = 1+x^{-2} = \boxed{1+\frac{1}{x^2}}$

c) $y' = 2-x^{-2}+6x^{-3}$

$\boxed{y' = 2-\frac{1}{x^2}+\frac{6}{x^3}}$

Review Set 20B continued

4) $y = 2x + 2x^{-1}$

$y' = 2 - 2x^{-2}$

$y' = 2 - \frac{2}{x^2}$

$0 = 2 - 2/x^2$

$\frac{2}{x^2} = 2$

$2 = 2x^2$

$1 = x^2$

$x = \pm 1$

$f(1) = 2(1) + 2(1)^{-1} = 4$

$f(-1) = 2(-1) + 2(-1)^{-1} = -4$

$\boxed{(1, 4) \quad (-1, -4)}$

6) Normal line:

Find y for $x = -3$

$(-3)^2 - 7(-3) - 44 = -14$

pt $(-3, -14)$

slope: $2x - 7$

$2(-3) - 7 = -13$

$m = 1/13$

$y + 14 = 1/13\,(x + 3)$

$y = 1/13\,(x+3) - 14$

Graph this and $y = x^2 - 7x - 44$ and see where else they cross in

$x \approx 10.1 \quad y \approx -13.0 \quad \boxed{(10.1, -13.0)}$

8) $y = 2x + 4x^{-2}$

$y' = 2 - 8x^{-3}$

$1 = 2 - 8x^{-3}$

$-1 = -8x^{-3}$

$-1 = \frac{-8}{x^3}$

$-x^3 = -8$

$x^3 = 8$

$x = 2$

$y = 2(2) + 4/2^2 = 5$

@ $(2, 5)$

b) pt$(2,5)$ $m = 1$

$y - 5 = 1(x - 2)$

$y = 1x - 2 + 5$

$\boxed{y = x + 3}$

c) cuts x-axis $\Rightarrow y = 0$

$0 = x + 3$

$x = -3$

$\boxed{(-3, 0)}$

d) pt$(2,5)$ $m = 1$

$y - 5 = -1(x - 2)$

$\boxed{y = -x + 7}$

Review Set 20C

1) a) $f'(x) = 4x^3 + 6x^2 + 6x$

b) $f'(x) = -6x^{-4} - 4x^{-5}$

$f'(x) = \frac{-6}{x^4} - \frac{4}{x^5}$

c) $f(x) = x^{-1} - 4x^{-2}$

$f'(x) = -x^{-2} + 8x^{-3}$

$f'(x) = \frac{-1}{x^2} + \frac{8}{x^3}$

2) d) $f'(x) = 3x^2 - 2x - 1$

$f'(0) = 3(0)^2 - 2(0) - 1 = \boxed{-1}$

4) $S'(t) = 0.9t^2 - 36t + 550$

This is the rate of change of the weight of sand at a given time t (instantaneous change)

6) $y' = 3x^2 - 4x + a$

at $x = 2$ $m = 7$

$7 = 3(2)^2 - 4(2) + a$

$7 = 12 - 8 + a$

$7 = 4 + a$

$\boxed{a = 3}$

plug $(2, -1)$ into y and $a = 3$

$-1 = (2)^3 - 2(2)^2 + (3)(2) - b$

$-1 = 8 - 8 + 6 - b$

$-1 = 6 - b$

$-7 = -b$

$\boxed{b = 7}$

8) a) $f'(x) = 3x^2 - 8x + 4$

b) $f'(1) = 3(1)^2 - 8(1) + 4 = 3 - 8 + 4 = \boxed{-1}$

* The gradient of the tangent line at $x = 1$ is -1 *

c) $x = 2$

$f'(2) = 3(2)^2 - 8(2) + 4 = 12 - 16 + 4 = \boxed{0}$

d) pt $(2, 1)$ $m = 0$

$y - 1 = 0(x - 2)$

$y - 1 = 0$

$\boxed{y = 1}$

Review Set 21A

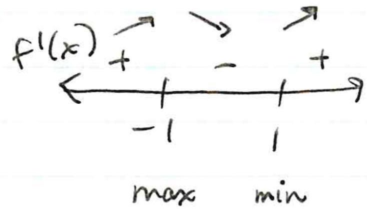
1) (a) increasing $(-\infty,-1)$ $(4,\infty)$ decreasing $(-1,4)$
(b) increasing $(-3,\infty)$ decreasing $(-\infty,-3)$
(c) increasing $(-\infty,6)$ decreasing never

2) (a) y-int $\Rightarrow x=0$

$y=(0)^3-3(0)=0$

(b) $f'(x) = 3x^2-3$

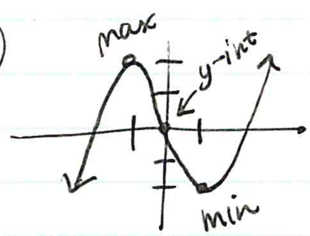
(c) $0 = 3x^2-3$
$3 = 3x^2$
$1 = x^2$
$x = \pm 1$



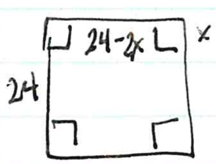
$f'(-2)=9$
$f'(0) = -3$
$f'(2) = 9$

$f(-1) = (-1)^3-3(-1)=2$ max $(-1,2)$
$f(1) = (1)^3-3(1)=-2$ min $(1,-2)$

(d)



4)



$V = x(24-2x)(24-2x)$
$x(576-96x+4x^2)$
$V = 576x-96x^2+4x^3$
$V' = 576-192x+12x^2$
$0 = 12x^2-192x+576$
$0 = 12(x^2-16x+48)$
$0 = 12(x-12)(x-4)$
$x=12 \quad x=4$

$\boxed{x=4}$

$\boxed{4cm \times 4cm}$

x = 12 gives side length zero

Review Set 21A continued

5) a) $f'(x) = 3x^2 - 12$

b) $x = -3$

$3(-3)^2 - 12$

$\boxed{15}$

c) $0 = 3x^2 - 12$

$12 = 3x^2$

$4 = x^2$

$x = 2$

(based on graph)

$f(2) = -12$

$\boxed{(2, -12)}$

6) a) Profit = Income − Cost

x = # packs of 1000

$= 28x - (0.4x^2 + 1.6x + 150)$

$= 28x - 0.4x^2 - 1.6x - 150$

$P = -0.4x^2 + 26.4x - 150$

$P' = -0.8x + 26.4$

$0 = -0.8x + 26.4$

$0.8x = 26.4$

$x = 33$ packs of 1000 or 33,000 chopsticks

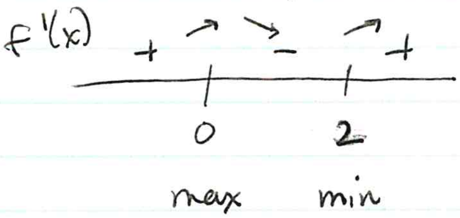
b) profit $= -0.4(33)^2 + 26.4(33) - 150 = \$285.60$

Review Set 21B

1) $f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$x = 0 \quad x = 2$



$f'(3) = 9$

$f'(1) = -3$

$f'(-1) = 9$

$f(x) = x^3 - 3x^2 + 5$

$f(0) = 5$

$f(2) = 1$

$f(-1) = 1$

$f(4) = 21$

$\boxed{\text{max} = 21 \quad \text{min} = 1}$

Review Set 21B continued

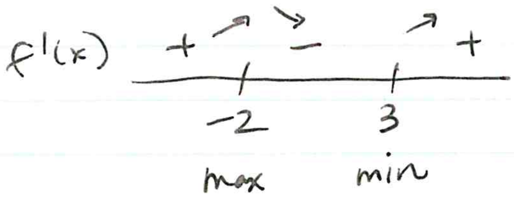
2) ⓐ $f'(x) = 6x^2 - 6x - 36$

ⓑ $0 = 6x^2 - 6x - 36$

$0 = 6(x^2 - x - 6)$

$0 = 6(x-3)(x+2)$

$x = 3 \quad x = -2$



$f'(-3) = 36$

$f'(0) = -36$

$f'(4) = 36$

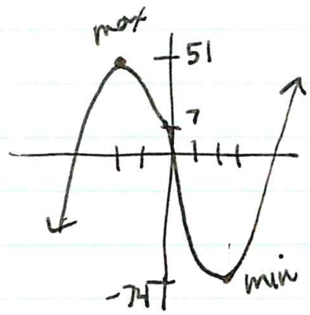
ⓑ $f(-2) = 51$ max $(-2, 51)$

$f(3) = -74$ min $(3, -74)$

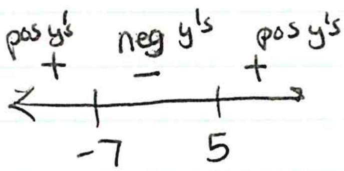
ⓒ increasing $(-\infty, -2)\ (3, \infty)$

decreasing $(-2, 3)$

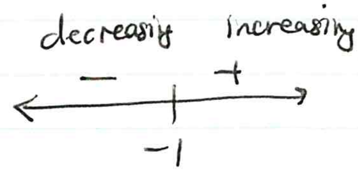
ⓓ



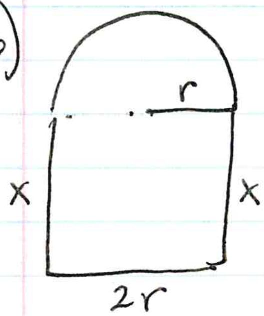
4) $f(x)$



$f'(x)$



6)



ⓐ $P = \frac{1}{2}(2\pi r) + x + x + 2r$

$P = \pi r + 2x + 2r$

ⓑ $200 = \pi r + 2x + 2r$

$200 - \pi r - 2r = 2x$

$100 - \frac{1}{2}\pi r - r = x$

ⓒ $A = \frac{1}{2}(\pi r^2) + 2r(x)$

$A = \frac{1}{2}\pi r^2 + 2r(100 - \frac{1}{2}\pi r - r)$

$= \frac{1}{2}\pi r^2 + 200r - \pi r^2 - 2r^2$

$= 200r - 2r^2 - \frac{1}{2}\pi r^2$

$200r - r^2(2 + \pi/2)$

Review set 21C

3) (a)(i) $v = 15 \text{ km h}^{-1}$

$10(15) + 90/15 = 150 + 6 = €156$ per hour

2 hours $= \boxed{€312}$

(c) $C(v) = 10v + 90v^{-1}$

$C'(v) = 10 - 90v^{-2}$

$0 = 10 - \frac{90}{v^2}$

$\frac{90}{v^2} = 10$

$90 = 10v^2$

$9 = v^2$

$v = \pm 3$ $\longrightarrow$ so $v = 3 \text{ km h}^{-1}$ (can't be negative)

8) (a) $f'(x) = 6x + 2$

$f'(2) = 6(2) + 2 = 14$

(b) Since $f'(2)$ is positive it is increasing at $x = 2$

9) (a) $(-\infty, -3)$ $(-1, 2)$ $(4, \infty)$

(b) $(-3, -1)$ $(2, 4)$

(c) same as a

(d) same as b

(e) $x = -3, -1, 2, 4$

(f) same as a + c

(g) same as b + d

(h) same as e

(i) Same as a, c, f

(j) Same as b, d, g

(k) local $(-3, 2)$ $(2, 1)$ $\rightarrow$ absolute $(-3, 2)$

(l) local $(-1, -3)$ $(4, -1)$ $\rightarrow$ absolute $(-1, -3)$