Solutions and Explanations for the Practice Exercises

EXPONENTS AND LOGARITHMS

1. Simplify each exponential expression to the form x^n where n is positive.
   Use the properties of exponents to simplify these expressions.

   (a) $\sqrt{\dfrac{x^3 \cdot x^8}{x^6}} = \sqrt{\dfrac{x^{11}}{x^6}} = \sqrt{x^5} = x^{5/2}$

   (b) $\left(\dfrac{x^{-2}}{x^{-6}}\right)^3 = \left(x^4\right)^3 = x^{12}$

   (c) $\left(\dfrac{1}{x^2}\right)^{-3} = \left(x^{-2}\right)^{-3} = x^6$

2. (No Calculator Allowed) Write each expression as a single logarithm.
   Use the laws of logarithms to simplify these expressions.

   (a) $\log 8 + \log 6 - \log 12 = \log(8 \cdot 6) - \log 12 = \log\left(\dfrac{48}{12}\right) = \log 4$

   This problem uses both the product law and the quotient law. Work from left to right when applying these two laws in the same problem.

   (b) $2\log 6 - \frac{1}{2}\log 9 = \log 6^2 - \log 9^{\frac{1}{2}} = \log 36 - \log\sqrt{9} = \log 36 - \log 3$

   $$= \log\left(\frac{36}{3}\right) = \log 12$$

   When using laws of logarithms to combine log expressions, always apply the power law first when it is needed. Then apply the quotient law.

   (c) $-3\log 2 + 2\log 4 = \log 2^{-3} + \log 4^2 = \log\frac{1}{8} + \log 16 = \log\left(\frac{1}{8} \cdot 16\right) = \log 2$

   Apply the power law first and then the product law.

3. (No Calculator Allowed) Let $x = \log_A 5$, $y = \log_A 2$, and $z = \log_A 3$. Write each expression in terms of x, y, and/or z.

   (a) $\log_A 12 = \log_A\left(2^2 \cdot 3\right) = \log_A 2^2 + \log_A 3 = 2\log_A 2 + \log_A 3 = 2y + z$

   First rewrite the argument, 12, in terms of 2 and 3, since these numbers are two of the arguments in the given log expressions. Then split the expression apart using the laws of logarithms. When using laws of logarithms to break down log expressions, apply the product and quotient laws first, working from left to right. Then apply the power law.

   (b) $\log_A 7.5 = \log_A\left(\dfrac{5 \cdot 3}{2}\right) = \log_A(5 \cdot 3) - \log_A 2 = \log_A 5 + \log_A 3 - \log_A 2 = x + z - y$

   Rewrite the argument, and then apply the laws of logarithms in the proper order.

   (c) $\log_5 3 = \dfrac{\log_A 3}{\log_A 5} = \dfrac{z}{x}$

   This problem uses the change of base formula.

4. (No Calculator Allowed) Solve for x: $27^{x-2} = \left(\frac{1}{9}\right)^{3x}$

$$\left(3^3\right)^{x-2} = \left(3^{-2}\right)^{3x}$$
$$3^{3x-6} = 3^{-6x}$$
$$3x - 6 = -6x$$
$$9x = 6$$
$$x = \frac{2}{3}$$

Start by rewriting the base on each side as a power of 3. Then apply the power of a power property, and multiply the exponents.

Now that the bases are the same on both sides, you can set the exponents equal to each other and solve for x.

5. Solve each logarithmic equation. Remember to check for extraneous solutions.

(a) $5\log_3 x - 2 = 8$

$$5\log_3 x = 10$$
$$\log_3 x = 2$$
$$3^2 = x$$
$$x = 9$$

First isolate the log expression on one side of the equation. Then use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in exponential form. Solve for x.

(b) $5\log_3 (x-2) = 8$

$$\log_3 (x-2) = 1.6$$
$$3^{1.6} = x - 2$$
$$x \approx 7.80$$

In this equation, the minus 2 is inside parentheses, meaning it is part of the argument of the log expression. Isolate the log expression by dividing both sides by 5. Then use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in exponential form. Solve for x. Round your answer to three significant figures.

(c) $\ln(5x) + \ln(2x) = 3$

$$\ln(10x^2) = 3$$
$$e^3 = 10x^2$$
$$x^2 = \frac{e^3}{10}$$
$$x = \pm\sqrt{\frac{e^3}{10}}$$

$x \approx 1.42$, ~~$x \approx -1.42$~~

Start by using the product law to combine the two log expressions. Then use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in exponential form. Remember that "ln" means natural log, and its base is the number e. Solve for x, and round your answer to three significant figures. The negative root is an extraneous answer in this problem because the argument of a logarithm cannot be negative.

(d) $\log(5x+1) - \log(x-2) = 1$

$$\log\left(\frac{5x+1}{x-2}\right)=1$$

$$\frac{5x+1}{x-2}=10^1$$

$$5x+1=10x-20$$

$$5x=21$$

$$x=4.2$$

Use the quotient law to combine the two log expressions. Then use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in exponential form. The base in this problem is 10. Then solve for x.

(e) $\log_2(x-5) + \log_2(x+1) = 4$

$$\log_2((x-5)(x+1))=4$$

$$\log_2\left(x^2-4x-5\right)=4$$

$$2^4 = x^2-4x-5$$

$$16 = x^2-4x-5$$

$$0 = x^2-4x-21$$

$$0=(x-7)(x+3)$$

$$x=7,\ \cancel{x=-3}$$

Use the product law to combine the two log expressions. Then use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in exponential form. Then solve for x. Negative 3 is an extraneous solution because substituting it back into the original equation gives $\log_2(-8)$ and $\log_2(-2)$, both of which are undefined.

6. Solve each exponential equation. Round answers to three significant figures.

(a) $3^{2x+1} = 5$

$$2x+1=\log_3 5$$

$$2x+1=\frac{\log 5}{\log 3}$$

$$2x+1 = 1.46497\ldots$$

$$x \approx 0.232$$

Use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in logarithmic form. Then solve for x. You may need to use the change of base formula to evaluate $\log_3 5$ on your calculator. Don't round until the final step to avoid rounding error.

(b) $2e^{3x} + 1 = 9$

$$2e^{3x}=8$$

$$e^{3x}=4$$

$$3x=\ln 4$$

$$x=\frac{\ln 4}{3}\approx 0.462$$

Start by isolating the exponential expression on one side of the equation. Then use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in logarithmic form. The base in this problem is e, so use natural log. Then solve for x.

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(c) $2^{3x-2} = 64$

$3x - 2 = \log_2 64$

$3x - 2 = 6$

$x = \frac{8}{3} \approx 2.67$

Use the definition of logarithms, $a^x = b \Leftrightarrow x = \log_a b$, to rewrite the equation in logarithmic form. Then solve for x.

7. (No Calculator Allowed) Let $f(x) = 4(2)^{x-1} + 3$.

(a) Find the coordinates of the y-intercept on the graph of f.

$f(0) = 4(2)^{0-1} + 3 = 5$

The y-intercept is (0, 5).

The y-intercept on a function graph is always the point at which x equals zero. Solve this problem by substituting zero for x in the given equation. Be sure to write your answer in the form of an ordered pair, (x, y).

(b) The graph of f passes through the point $(k, 35)$. Find the value of k.

$35 = 4(2)^{k-1} + 3$

$32 = 4(2)^{k-1}$

$8 = (2)^{k-1}$

$2^3 = (2)^{k-1}$

$k - 1 = 3$

$k = 4$

Substitute 35 for y and k for x in the given equation. Then solve for k. Be sure to follow the order of operations correctly!

SEQUENCES AND SERIES

1. Classify each sequence as arithmetic or geometric. Then find an expression for the general term of each sequence. Finally, find the 10th term in each sequence.

(a) 3, 7, 11, 15, . . .

This sequence is arithmetic.

Solve the equation below to find the common difference:

$$3 + d = 7$$
$$d = 4$$

Then check that adding 4 each time gives you the next term in sequence.

An expression for the general term of the sequence is $u_n = 3 + (n - 1)4$

Use the formula $u_n = u_1 + (n - 1)d$ to find the general term. Note that you are not required to simplify your answer unless you are specifically asked to do so.

The 10th term is $u_{10} = 3 + (10 - 1)4 = 39$

Substitute $n = 10$ to find the tenth term.

(b) 6.2, 8, 9.8, 11.6, . . .

This sequence is arithmetic.

Solve the equation below to find the common difference:

$$6.2 + d = 8$$
$$d = 1.8$$

Then check that adding 1.8 each time gives you the next term in sequence.

An expression for the general term of the sequence is $u_n = 6.2 + (n - 1)(1.8)$

Use the formula $u_n = u_1 + (n - 1)d$ to find the general term.

The 10th term is $u_{10} = 6.2 + (10 - 1)(1.8) = 22.4$

Substitute $n = 10$ to find the 10th term.

(c) $\frac{1}{3}$, 1, 3, 9, . . .

This sequence is geometric.

Solve the equation below to find the common ratio:

$$\frac{1}{3}r = 1$$
$$r = 3$$

Then check that multiplying by 3 each time gives you the next term in sequence.

An expression for the general term of the sequence is $u_n = \frac{1}{3} \cdot 3^{n-1}$

Use the formula $u_n = u_1 \cdot r^{n-1}$ to find the general term.

The 10th term is $u_{10} = \frac{1}{3} \cdot 3^{10-1} = 6561$

Substitute $n = 10$ to find the 10th term.

(d) 23, 21, 19, 17, . . .

This sequence is arithmetic.

Solve the equation below to find the common difference:

$$23 + d = 21$$
$$d = -2$$

Then check that adding –2 each time gives you the next term in sequence.

An expression for the general term of the sequence is $u_n = 23 + (n - 1)(-2)$

Use the formula $u_n = u_1 + (n - 1)d$ to find the general term.

The 10th term is $u_{10} = 23 + (10 - 1)(-2) = 5$

Substitute $n = 10$ to find the 10th term.

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(e) 8, 12, 18, 27, . . .

This sequence is geometric.

Solve the equation below to find the common ratio:

$$8r = 12$$
$$r = 1.5$$

Then check that multiplying by 1.5 each time gives you the next term in sequence.

An expression for the general term of the sequence is $u_n = (8)(1.5)^{n-1}$

Use the formula $u_n = u_1 \cdot r^{n-1}$ to find the general term.

The 10th term is
$u_{10} = (8)(1.5)^{10-1}$
$= 307.546... \approx 308$

Substitute $n = 10$ to find the 10th term. When rounding is necessary, always round to three significant figures.

(f) 960, 288, 86.4, 25.92, . . .

This sequence is geometric.

Solve the equation below to find the common ratio:

$$960r = 288$$
$$r = 0.3$$

Then check that multiplying by 0.3 each time gives you the next term in sequence.

An expression for the general term of the sequence is $u_n = 960(0.3)^{n-1}$

Use the formula $u_n = u_1 \cdot r^{n-1}$ to find the general term.

The 10th term is
$u_{10} = 960(0.3)^{10-1}$
$= 0.01889... \approx 0.0189$

Substitute $n = 10$ to find the 10th term. Round to three significant figures.

2. Find the sum of the first 15 terms of each series.

(a) $\frac{1}{2} + 2 + \frac{7}{2} + 5 + \ldots$

$\frac{1}{2} + d = 2$

First determine the type of series. This series is arithmetic.

$d = \frac{3}{2}$

$S_{15} = \frac{15}{2}\left(2 \cdot \frac{1}{2} + (15-1)\left(\frac{3}{2}\right)\right) = 165$

Then use the formula $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ to find the 15th partial sum.

(b) $2 + 4 + 8 + 16 + \ldots$

$2r = 4$

$r = 2$

$S_{15} = \frac{2(1-2^{15})}{1-2} = 65{,}534$

This series is geometric.

Use the formula $S_n = \frac{u_1(1-r^n)}{1-r}$ to find the 15th partial sum.

(c) $15 + 11 + 7 + 3 + \ldots$

$15 + d = 11$

$d = -4$

$S_{15} = \frac{15}{2}(2(15) + (15-1)(-4)) = -195$

This series is arithmetic.

Use the formula $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ to find the 15th partial sum.

3. Find the sum of the given series. (Note that you must first calculate the number of terms.)

(a) $3.7 + 4.2 + 4.7 + \ldots + 27.2$

$3.7 + d = 4.2$

$d = 0.5$

First determine the type of series. This sequence is arithmetic.

$27.2 = 3.7 + (n-1)(0.5)$

$23.5 = 0.5n - 0.5$

$24 = 0.5n$

$n = 48$

Find the number of terms in the series by using the formula $u_n = u_1 + (n-1)d$ to find the value of n that corresponds to the last term shown. Remember that n is the term number, so it must always be a whole number.

$S_{48} = \frac{48}{2}(3.7 + 27.2) = 741.6$

Then use the formula $S_n = \frac{n}{2}(u_1 + u_n)$ to find the sum of the series.

(b) $80 + 74 + 68 + \ldots + 14$

$80 + d = 74$
$d = -6$

This sequence is arithmetic.

$14 = 80 + (n-1)(-6)$
$-66 = -6n + 6$
$-72 = -6n$
$n = 12$

Use the formula $u_n = u_1 + (n-1)d$ to find the number of terms in the series.

$S_{12} = \frac{12}{2}(80 + 14) = 564$

Then use the formula $S_n = \frac{n}{2}(u_1 + u_n)$ to find the sum of the series.

(c) $0.125 + 0.25 + 0.5 + \ldots + 64$

$0.125r = 0.25$
$r = 2$

This series is geometric.

$64 = (0.125)2^{n-1}$

$512 = 2^{n-1}$

$n - 1 = \log_2 512$

$n = \frac{\log 512}{\log 2} + 1 = 10$

Use the formula $u_n = u_1 \cdot r^{n-1}$ to find the number of terms in the series.

$S_{10} = \frac{0.125\left(1-2^{10}\right)}{1-2} = 127.875$

Then use the formula $S_n = \frac{u_1\left(1-r^n\right)}{1-r}$ to find the sum.

4. In an arithmetic sequence, $u_{15} = 25.4$ and $u_{28} = 46.2$.

(a) Find an expression for the general term.

$25.4 = u_1 + (15-1)d$
$46.2 = u_1 + (28-1)d$

We know that the 15th term is 25.4 and the 28th term is 46.2. Plug each of these pairs of values into the formula for a term in an arithmetic sequence, $u_n = u_1 + (n-1)d$.

$$\begin{array}{r} 25.4 = u_1 + 14d \\ -(46.2 = u_1 + 27d) \\ \hline -20.8 = -13d \end{array}$$

$d = 1.6$
$25.4 = u_1 + 14(1.6)$
$25.4 = u_1 + 22.4$
$u_1 = 3$

Now we have a system of equations that we can solve to find u_1 and d. Simplify each equation, and subtract them to eliminate u_1. Then solve for d. Substitute for d in one of the original equations, and solve for u_1.

$u_n = 3 + (n-1)(1.6)$

Then use the formula $u_n = u_1 + (n-1)d$ to find the general term.

(b) Find the first term to exceed 100.

$100 = 3 + (n-1)(1.6)$ $97 = (n-1)(1.6)$ $60.625 = n-1$ $n = 61.625$	Use the answer from part (a), and plug in 100 for the term u_n. Solve for n. This means that the 62nd term is the first term to exceed 100. (Remember that n can be only a whole number, so you need to round up!)
$u_{62} = 3 + (62-1)(1.6) = 100.6$	Plug into the formula $u_n = u_1 + (n-1)d$ again to find the 62nd term.

5. (No Calculator Allowed) Consider two geometric series given by $\sum_{r=1}^{n} 6\cdot\left(\frac{1}{2}\right)^r$ and $\sum_{r=1}^{n} 6\cdot\left(\frac{3}{2}\right)^r$.

(a) Write down the first four terms of each series. Simplify your answers, but leave them exact.

$$\sum_{r=1}^{4} 6\cdot\left(\tfrac{1}{2}\right)^r = 6\left(\frac{1}{2}\right)^1 + 6\left(\frac{1}{2}\right)^2 + 6\left(\frac{1}{2}\right)^3 + 6\left(\frac{1}{2}\right)^4$$
$$= \frac{6}{2} + \frac{6}{4} + \frac{6}{8} + \frac{6}{16}$$
$$= 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8}$$

To find the first four terms, plug in $r = 1, 2, 3$, and 4. Remember sigma notation means that the terms are written as sums.

$$\sum_{r=1}^{4} 6\cdot\left(\tfrac{3}{2}\right)^r = 6\left(\frac{3}{2}\right)^1 + 6\left(\frac{3}{2}\right)^2 + 6\left(\frac{3}{2}\right)^3 + 6\left(\frac{3}{2}\right)^4$$
$$= \frac{18}{2} + \frac{54}{4} + \frac{162}{8} + \frac{486}{16}$$
$$= 9 + \frac{27}{2} + \frac{81}{4} + \frac{243}{8}$$

(b) One of these series can be summed to infinity. Find this infinite sum.

The series $\sum_{r=1}^{n} 6\cdot\left(\frac{3}{2}\right)^r$ has $r = \frac{3}{2}$. The series $\sum_{r=1}^{n} 6\cdot\left(\frac{1}{2}\right)^r$ has $r = \frac{1}{2}$.	Start by determining the common ratio, r, for each series. A geometric series can be summed to infinity only when $\lvert r\rvert < 1$.
$S_\infty = \dfrac{3}{1-\frac{1}{2}} = 3 \div \dfrac{1}{2} = 6$	The series $\sum_{r=1}^{n} 6\cdot\left(\frac{1}{2}\right)^r$ can be summed to infinity using the formula $S_\infty = \dfrac{u_1}{1-r}$.

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6. The current population of a city is 275,000 people, and the population is growing at a constant rate of 3.1% annually.

(a) Write an expression for the population at the end of n full years.

The population after n years is $275000(1.031)^n$

Each year, 3.1% is added to the current population. So you need to multiply by 103.1% or 1.031. Start by writing expressions for the population during the first few years. Then look for a pattern.

Current year: 275000

After 1 year: $275000(1.031)$

After 2 years: $275000(1.031)(1.031) = 275000(1.031)^2$

After 3 years: $275000(1.031)^2(1.031) = 275000(1.031)^3$

After n years: $275000(1.031)^n$

(b) How many full years are required for the population to double?

$2(275000) = 550000$

First find double the current population.

$550000 = 275000(1.031)^n$

$2 = (1.031)^n$

$n = \log_{1.031} 2$

$n = \dfrac{\log 2}{\log 1.031} \approx 22.7$

Set your expression from part (a) equal to this value, and solve for n.

The population will be doubled at the end of 23 full years.

The problem asks how many *full* years, so round up to the nearest whole year.

7. (No Calculator Allowed) The first three terms of an arithmetic sequence are $2x-5$, $x-1$, and $2x$. Find the value of x.

$2x-5+d=x-1$

$x-1+d=2x$

Since this series is arithmetic, we know that each new term can be obtained from the previous term by adding by the same number. Use d to represent the number we add, and write two equations.

$d=x-1-(2x-5)$

$d=2x-(x-1)$

$x-1-(2x-5)=2x-(x-1)$

$x-1-2x+5=2x-x+1$

$-x+4=x+1$

$2x=3$

$x=1.5$

Now we solve each of these equations for d, and set them equal to each other. Then we can solve the resulting equation for x.

The Binomial Theorem

1. Write out the expansion of each binomial. Simplify your answers.
   (a) There are five terms in this expansion. Use the properties of the binomial expansion to write out the terms, and then simplify. Don't forget to put parentheses around the –3!
   $(x-3)^4 = 1 \cdot x^4 \cdot (-3)^0 + 4 \cdot x^3 \cdot (-3)^1 + 6 \cdot x^2 \cdot (-3)^2 + 4 \cdot x^1 \cdot (-3)^3 + 1 \cdot x^0 \cdot (-3)^4$
   $= x^4 - 12x^3 + 54x^2 - 108x + 81$
   (b) This expansion has six terms. Here parentheses are required around the $2x$ as well as the –1.
   $(2x-1)^5$
   $= 1(2x)^5(-1)^0 + 5(2x)^4(-1)^1 + 10(2x)^3(-1)^2 + 10(2x)^2(-1)^3 + 5(2x)^1(-1)^4 + 1(2x)^0(-1)^5$
   $= 32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$
   (c) This expansion has eight terms.
   $(p+q)^7 = p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4 + 21p^2q^5 + 7pq^6 + q^7$
   (d) This expansion has five terms.
   $(x-h)^4 = x^4 - 4x^3h + 6x^2h^2 - 4xh^3 + h^4$

2. Consider the expression $(2x-5)^8$.
   (a) Write down the number of terms in the expansion of this expression.
   The number of terms is always one more than the exponent, so this expansion has 9 terms.
   (b) Find the 6th term in the expansion of this expression.
   Use the formula $\binom{n}{r} a^{n-r}b^r$, with $n = 8$, $a = 2x$, $b = -5$, and $r = 5$.
   $$\binom{8}{5}(2x)^{8-5}(-5)^5 = 56 \cdot 2^3 \cdot x^3 \cdot (-3125) = -1{,}400{,}000x^3$$

3. (No Calculator Allowed) Find the term containing p^7 in the expansion of $(p+q)^{10}$.
   Use the formula $\binom{n}{r} a^{n-r}b^r$, with $n = 10$, $a = p$, and $b = q$. After substituting these three values, we have $\binom{10}{3} p^{10-r}q^r$. Now we can see that r must equal 3. Calculators are not allowed, so use the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ to find $\binom{10}{3}$.
   $$\binom{10}{3} p^{10-3}q^3 = \frac{10!}{3! \cdot 7!} p^7q^3 = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!} p^7q^3 = (10 \cdot 3 \cdot 4)p^7q^3 = 120p^7q^3$$

4. Find the values of p and q in the binomial expansion shown here:
   $(3x+p)^6 = 729x^6 - 486x^5 + qx^4 + \ldots$
   Start by using the properties of the binomial expansion to write out the first three terms in the expansion of $(3x+p)^6$. Remember to put parentheses around $3x$.
   $(3x+p)^6 = 1(3x)^6p^0 + 6(3x)^5p^1 + 15(3x)^4p^2 + \ldots$
   $= 729x^6 + 1458px^5 + 1215p^2x^4 + \ldots$
   Now we can equate these terms with the terms given in the question:
   $729x^6 + 1458px^5 + 1215p^2x^4 + \ldots = 729x^6 - 486x^5 + qx^4 + \ldots$

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Look at the second terms on each side. We can see that $1458p = -486$. Solving for p gives $p = -\frac{1}{3}$. Now use the third terms on each side to solve for q:

$q = 1215p^2 = 1215\left(-\frac{1}{3}\right)^2 = 135.$

5. (No Calculator Allowed) Find the constant term in the expansion of $(3x+2)^2\left(1+\frac{1}{x}\right)^4$.
Start by expanding each of the two binomial powers:
$(3x+2)^2 = 9x^2 + 12x + 4$

$$\left(1+\frac{1}{x}\right)^4 = 1(1)^4\left(\frac{1}{x}\right)^0 + 4(1)^3\left(\frac{1}{x}\right)^1 + 6(1)^2\left(\frac{1}{x}\right)^2 + 4(1)^1\left(\frac{1}{x}\right)^3 + 1(1)^0\left(\frac{1}{x}\right)^4$$

$$= 1 + \frac{4}{x} + \frac{6}{x^2} + \frac{4}{x^3} + \frac{1}{x^4}$$

Now we see that $(3x+2)^2\left(1+\frac{1}{x}\right)^4 = (9x^2 + 12x + 4)\left(1+\frac{4}{x}+\frac{6}{x^2}+\frac{4}{x^3}+\frac{1}{x^4}\right)$. The next step in expanding this expression would be to distribute every term in the first expansion to every term in the second expansion. However, we don't need to write out all of these terms, because we need to find only the constant term, that is, the term that does not contain x. The constant term is:

$$9x^2\left(\frac{6}{x^2}\right) + 12x\left(\frac{4}{x}\right) + 4(1) = 9(6) + 12(4) + 4 = 54 + 48 + 4 = 106.$$

FEATURED QUESTION (ANSWER)

May 2011, Paper 2

Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$.
Start with the given expression, and use laws of logarithms to combine the terms. Since this is a "show that" problem, make sure to write out every step!

$$f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$$
$$= \log_3\left(\frac{x}{2}\cdot 16\right) - \log_3 4$$
$$= \log_3 8x - \log_3 4$$
$$= \log_3 \frac{8x}{4}$$
$$= \log_3 2x$$

(b) Find the value of $f(0.5)$ and of $f(4.5)$.
Substitute the x-values into the function and simplify.
$f(0.5) = \log_3 2(0.5) = \log_3 1 = 0$
$f(4.5) = \log_3 2(4.5) = \log_3 9 = 2$

The function f can also be written in the form $f(x) = \frac{\ln ax}{\ln b}$.

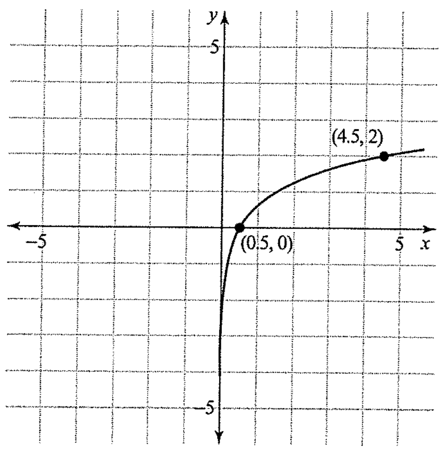
(c) (i) Write down the value of a and of b.

This question uses the change of base formula. Be sure to label your answers!

$f(x) = \log_3 2x = \dfrac{\ln 2x}{\ln 3}$, so $a = 2$ and $b = 3$.

(ii) Hence on graph paper, sketch the graph of f for $-5 \le x \le 5$, $-5 \le y \le 5$, using a scale of 1 cm to 1 unit on each axis.

This problem says "hence," so be sure to plot accurately the two points from earlier in the question. Also make sure the graph is drawn on exactly the specified intervals.



(iii) Write down the equation of the asymptote.

$x = 0$ The asymptote is clear from the graph. Just make sure to write an equation!

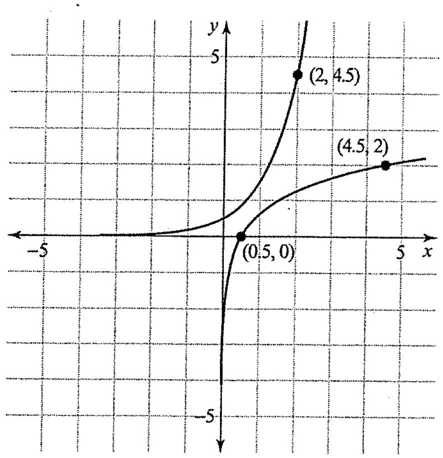
(d) Write down the value of $f^{-1}(0)$.

$f^{-1}(0) = 0.5$ We already found $f(0.5) = 0$, so just switch x and y to get the value of $f^{-1}(0)$.

The point A lies on the graph of f. At A, $x = 4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A.

Plot the inverse function by switching the x-values and y-values of points on the graph of f. The image of A is the point (2, 4.5), so be sure this point is plotted clearly.



FEATURED QUESTION (ANSWER)

May 2006, Paper 2

Consider the geometric sequence −3, 6, −12, 24, . . .

(a) (i) Write down the common ratio.

$r = -2$ The common ratio, r, is the number that a term is multiplied by to get the next term in the sequence. So you can calculate r by solving the equation $-3r = 6$.

(ii) Find the 15th term.

Just use the formula for a term in a geometric sequence, $u_n = u_1 \cdot r^{n-1}$.

$u_{15} = -3(-2)^{15-1} = -49{,}152$

Consider the sequence $x - 3$, $x + 1$, $2x + 8$, . . .

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms.

All you need to do here is substitute 5 for x and simplify.

2, 6, 18, . . .

(ii) Find the common ratio.

$r = 3$ Calculate r by solving the equation $2r = 6$.

(c) Find the other value of x for which the sequence is geometric.

$(x-3)r = x+1$

$(x+1)r = 2x+8$

$r = \frac{x+1}{x-3}, \; r = \frac{2x+8}{x+1}$

$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$

$(x+1)^2 = (2x+8)(x-3)$

$x^2 + 2x + 1 = 2x^2 + 8x - 6x - 24$

$x^2 - 25 = 0$

$x^2 = 25 \rightarrow x = \pm 5$

The other value is $x = -5$.

Use the fact that the sequence is geometric to write two equations in terms of x and r. Then solve each equation for r, set the equations equal, and solve for x.

(d) For this value of x, find

(i) The common ratio;

−8, −4, −2, . . . Write out the first three terms by substituting $x = -5$.

$r = \frac{1}{2}$ Then solve $-8r = -2$ to find r.

(ii) The sum of the infinite sequence.

$\frac{-8}{1-\frac{1}{2}} = -16$ Use the infinite sum formula, $S_\infty = \frac{u_1}{1-r}$