9.7 Definite integrals with linear motion and other problems

Recall: **Displacement** (position) = s(t). V**elocity** v(t) = s'(t) and **acceleration** a(t) = v'(t) = s"(t). **Displacement** function tells us the distance and direction a particle is from origin at time, t.

$\int_{t_1}^{t_2} v(t)dt = s(t_2) - s(t_1)$ is the change in displacement from t_1 to t_2.

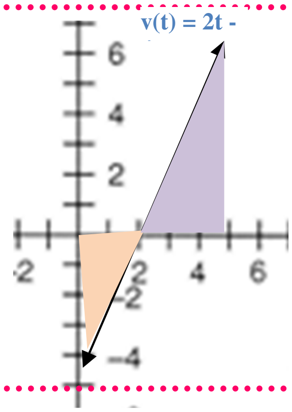
For example: $\int_0^5 v(t)dt = s(5) - s(0) = 8 - 3 = +5$. This means that at 5 seconds the particle is 5 meters to the right of where it was at 0 seconds (not necessarily the origin).

If v is the velocity function for a particle moving along a line, the **<u>total distance</u>** traveled from t_1 to t_2 is given by: Distance = $\int_{t_1}^{t_2} |v(t)|dt$.

mini example: Consider v(t) = 2t - 4

$\int_0^5 v(t)dt$ = -4 + 9 = 5

This gives the displacement from time 0 to 5 seconds.



$\int_0^5 |v(t)|dt = |-4| + 9 = 13$.

This gives the total distance of 13 meters traveled from 0 to 5 seconds.

Example 17: The displacement function for a particle moving along a horizontal line is given by $s(t) = 8 + 2t - t^2$ for $t \geq 0$, where t is measured in seconds and s is measured in meters.

a.) Find the velocity of the particle at time t. $s'(t) = v(t) = 2 - 2t$

b.) Find when the particle is moving right and when it is moving left.

$v(t) = 0 \begin{cases} 2 - 2t = 0 \\ t = 1 \end{cases}$ + | − (1) plug into v(t)

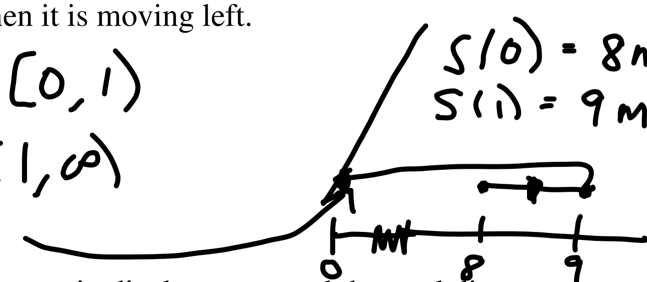
Right [0, 1)

Left (1, ∞)

s(0) = 8m

s(1) = 9m

c.) Draw a motion diagram for the particle.



d.) Write definite integrals to find the particle's change in displacement and the total distance traveled on the interval [0,4]. Use a GDC to evaluate the integrals and then use the motion diagram to verify results. s(4) = 0

Displacement

$\int_0^4 v(t)dt$

$\int_0^4 2 - 2t$

−8m

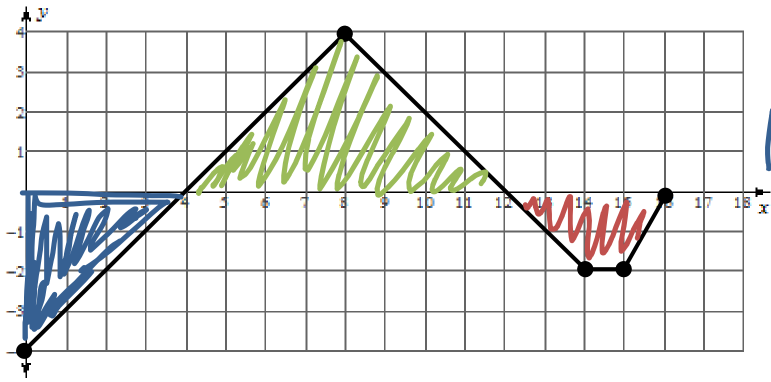
Distance

$\left|\int_0^1 v(t)dt\right| + \left|\int_1^4 v(t)dt\right|$

$|1 - 0| + |-8 - 1| = 10$

$\boxed{\text{or}}$ $\int_0^4 |v(t)|dt = \int_0^4 |2 - 2t|dt = 10$

Example 18: The velocity function v, in ms^{-1}, of a particle moving along a line is shown in the figure. Find the particle's change in displacement and the total distance traveled on the interval [0, 16].



Displacement

$-\frac{1}{2}(4)(4)$

$+\frac{1}{2}(8)(4)$

$-\frac{1}{2}(2)(4+1)$

$-8+16-5$

$\boxed{3\text{ m}}$

$\int_0^{16} v(t)\,dt$

Distance

$|-8|+|16|+|-5|$

29 m

$\int_0^{16} |v(t)|\,dt$

Hmuk 0

Example 19: A culture of bacteria is started with an initial population of 100 bacteria. The rate at which the number of bacteria changes over a one-month period can be modeled by the function $r(t) = e^{0.273t}$, where r is measured in bacteria per day. Find the population of bacteria 20 days after the culture was started.

$$\int_0^{20} r(t)\,dt = R(20) - R(0)$$

100 bacteria

$$\int_0^{20} r(t)\,dt = R(20) - 100$$

HmukP

$$100 + \int_0^{20} e^{0.273t} = R(20)$$

$$100 + \left[\frac{1}{.273} e^{.273t}\right]_0^{20} = R(20)$$

$$100 + \frac{1}{.273}\left(e^{.273(20)} - 1\right) = 957.4997 \approx 957$$

bacteria