9.4 Fundamental Theorem of Calculus

Fundamental Theorem of Calculus:

If f is a continuous function on the interval [a,b] and F is an antiderivative of f on [a,b], then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$.

Example 11: Evaluate the definite integral without a GDC.

a.) $\int_{-2}^{1}(u-1)du$

$\frac{1}{2}u^2 - u\Big]_{-2}^{1}$

$\left(\frac{1}{2}(1)^2 - 1\right) - \left(\frac{1}{2}(-2)^2 - (-2)\right)$

$-\frac{1}{2} - 4 =$ **-4.5**

b.) $\int_2^3 \frac{1}{t}dt$

$\ln|t|\Big]_2^3$

$\ln 3 - \ln 2$

$\ln 3/2$

c.) $\int_1^3 4x^2(x-1)dx$

$\int_1^3 (4x^3 - 4x^2)dx$

$x^4 - \frac{4}{3}x^3\Big]_1^3$

$\left(3^4 - \frac{4}{3}(3)^3\right) - \left(1^4 - \frac{4}{3}(1)^3\right)$

$(81 - 36) - \left(1 - \frac{4}{3}\right)$

$81 - 36 - 1 + \frac{4}{3} =$ **$45\frac{1}{3}$**

d.) $\int_0^2 (x^2+1)dx$

$\frac{1}{3}x^3 + x\Big]_0^2$

$\left(\frac{1}{3}(2)^3 + 2\right) - (0)$

$\frac{8}{3} + 2 =$ **$4\frac{2}{3}$ or $\frac{14}{3}$**

Example 12: Evaluate the definite integral without a GDC.

a.) $\int_1^5 \left(e^{2x} + \frac{1}{x^2}\right)dx$ ← save for last *

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

if $x = 1$, $u = 2(1) = 2$

if $x = 5$, $u = 2(5) = 10$

$\int_1^5 e^{2x}dx + \int_1^5 x^{-2}dx$

$\frac{1}{2}\int_2^{10} e^u du + \int_1^5 x^{-2}dx$

$\frac{1}{2}e^u\Big]_2^{10} + -x^{-1}\Big]_1^5$

$\frac{1}{2}e^u\Big]_2^{10} + -\frac{1}{x}\Big]_1^5$

$\frac{1}{2}\left(e^{10} - e^2\right) + \left(-\frac{1}{5} - \left(-\frac{1}{1}\right)\right)$

$\frac{1}{2}(e^{10} - e^2) + \frac{4}{5}$

b.) $\int_{-1}^{1}(2x-3)^3dx$

$\int_{-5}^{-1} u^3 \cdot \frac{1}{2}du$

$\frac{1}{2}\int_{-5}^{-1} u^3 du$

$\frac{1}{2}\cdot\frac{1}{4}u^4\Big]_{-5}^{-1}$

$\frac{1}{8}u^4\Big]_{-5}^{-1}$

$\frac{1}{8}\left[(-1)^4 - (-5)^4\right] = \frac{1}{8}[1 - 625)$

$-\frac{624}{8} =$ **-78**

$u = 2x - 3$

$du = 2dx$

$\frac{1}{2}du = dx$

if $x = -1$, $u = 2(-1) - 3 = -5$

if $x = 1$, $u = 2(1) - 3 = -1$

c.) $\int_0^3 \sqrt{3x+16}dx$

$u = 3x + 16$

$du = 3dx$

$\frac{1}{3}du = dx$

if $u = 0$, $3(0) + 16 = 16$

if $u = 3$, $3(3) + 16 = 25$

$\int_0^3 (3x+16)^{\frac{1}{2}}dx$

$\int_{16}^{25} (u)^{1/2} \cdot \frac{1}{3}du$

$\frac{1}{3}\int_{16}^{25} u^{1/2}$

$\frac{1}{3}\cdot\frac{2}{3}u^{3/2}\Big]_{16}^{25}$

$\frac{2}{9}u^{3/2}\Big]_{16}^{25}$

$\frac{2}{9}\left[\left((25)^{\frac{1}{2}}\right)^3 - \left((16)^{\frac{1}{2}}\right)^3\right]$

$\frac{2}{9}[125 - 64] =$ **$\frac{122}{9}$**

d.) $\int_0^1 (2x^2+1)^3(4x)dx$

$u = 2x^2 + 1$

$du = 4xdx$

if $x = 0$, $u = 2(0)^2 + 1 = 1$

if $x = 1$, $u = 2(1)^2 + 1 = 3$

$\int_1^3 (u)^3 du$

$\frac{1}{4}u^4\Big]_1^3$

$\frac{1}{4}\left[3^4 - 1^4\right]$

$\frac{1}{4}[81 - 1]$

$\frac{1}{4}(80) =$ **20**