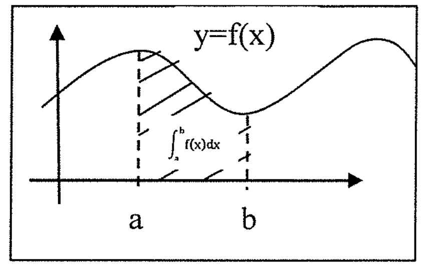
9.3 *Area and Definite Integrals (2 days)*

Sometimes we can't always evaluate integrals and need to use an approximation.

Do the INVESTIGATION on page 303

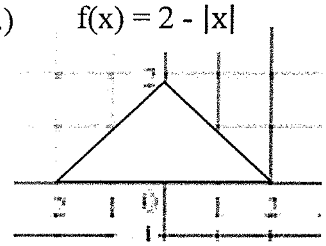
As you learned through your investigation in problem 1, you approximated the areas under curves by summing area together. Notice, that one answer gave you an **over** estimate, while one answer gave you an **under** estimate. Hence, we can denote integrals in terms of sums.

Definition: If f is **integrable** on an interval [a,b], we can call the limit the **definite integral** and denote it as $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$ where a is called the **lower limit** and b is called the **upper limit** of integration.



Example 8: Write down a definite integral that gives the area of the shaded region and evaluate it using a GDC (graphing calc). Whenever possible, find the area using a geometric formula to verify your answer.

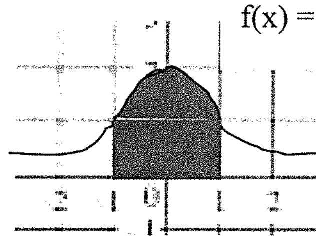
a.) f(x) = 2 - |x|



$\int_{-2}^{2} 2-|x|\,dx$

$A = \frac{1}{2}(4)(2) = 4$

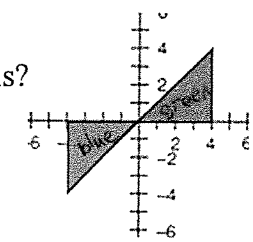
b.) $f(x) = \frac{2}{1+x^2}$



$\int_{-1}^{1} \frac{2}{1+x^2}\,dx \approx 3.14$

What if the function drops below the x-axis?

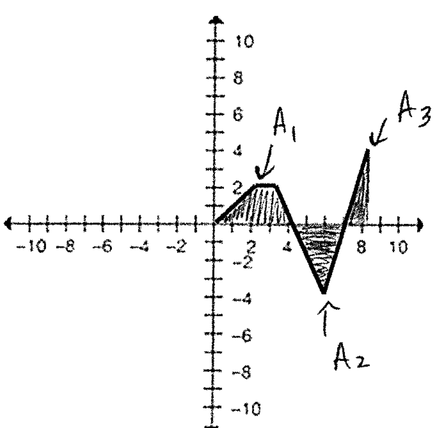
You must do the area in pieces…



Area of blue = 8 (below *x*-axis)
Area of green = 8
Total area = 16

Value of the integral: –8 + 8 = 0

Example 9: The graph of f consists of line segments show in the figure. Evaluate $\int_0^8 f(x)dx$ using geometric formulae.



$A_1 = \frac{1}{2}\cdot 2(4+1) = 5$

$A_2 = \frac{1}{2}\cdot(3)(4) = 6$ (below)

$A_3 = \frac{1}{2}(1)(4) = 2$

$\int_0^8 f(x)\,dx = 5 - 6 + 2 = \boxed{1}$

Hmwk G

Properties of definite integrals

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$

$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

$\int_a^a f(x)dx = 0$

$\int_a^b f(x)dx = -\int_b^a f(x)dx$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Example 10: Given that $\int_0^2 f(x)dx = 4$, $\int_2^5 f(x)dx = 12$, $\int_0^2 g(x)dx = -3$ and $\int_0^4 g(x)dx = 6$, evaluated these definite integrals without using your GDC.

a.) $\int_0^2 (3f(x)dx - g(x))dx$

$\int_0^2 3f(x)dx - \int_0^2 g(x)dx$

$3\int_0^2 f(x)dx - \int_0^2 g(x)dx$

$3(4) - (-3) = \boxed{15}$

b.) $\int_2^2 g(x)dx + \int_5^2 f(x)dx$

$6 + -\int_2^5 f(x)dx$

$-(12) = \boxed{-12}$

c.) $\int_0^5 f(x)dx$

$\int_0^2 f(x)dx + \int_2^5 f(x)dx$

$4 + 12$

16

d.) $\int_2^4 g(x)dx$

$\int_0^4 g(x)dx - \int_0^2 g(x)dx$

$6-(-3)$

9

e.) $\int_{-3}^{-1} \frac{1}{2}f(x+3)dx$

$\frac{1}{2}\int_{-3+3}^{-1+3} f(x)dx$

$\frac{1}{2}\int_0^2 f(x)dx$

$\frac{1}{2}(4)$

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