# *9.2 More on Indefinite Integrals (1 day notes, 2 days practice)*

*Be careful not to use power rule in every instance of $x^n$!

**More Integration Rules**

$\log_2 -8$

$\log_2 0$

- **Natural Log Rule:** $\int \frac{1}{x} dx = \ln|x| + C$ where $x > 0$
- **Base e Rule:** $\int e^x dx = e^x + C$

**Example 4:** Find the indefinite integral of the following.

a.) $\int \frac{4}{x} dx$

$4\int \frac{1}{x} dx$

$4 \ln|x| + C$

$x > 0$

b.) $\int \frac{e^t}{2} dt$

$\frac{1}{2}\int e^t dt$

$\frac{1}{2}e^t + C$

c.) $\int \frac{1}{4x} dx$

$\frac{1}{4}\int \frac{1}{x} dx$

$\frac{1}{4}\ln|x| + C$

**Example 5:** Find the indefinite integral.

a.) $\int (x^2+1)^2 dx$

$\int (x^4 + 2x^2 + 1) dx$

$\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$

b.) $\int \frac{3x^2+2x+1}{x} dx$

$\int \left(3x + 2 + \frac{1}{x}\right) dx$

$\frac{3}{2}x^2 + 2x + \ln|x| + C$

c.) $\int \cancel{\ln}(\cancel{e}^{2t-1})\, dt$

$\int (2t-1) dt$

$t^2 - t + C$

Hmwk D

**More Integration Rules**

*Note: I'm skipping the first way your book teaches you. See page 298 if interested*

**Substitution Method:**

look for ways to change your integral into the form $\int f(g(x))g'(x)dx$

*Note: If you'd need chain rule to take the derivative, you need to substitute for integration.

**Example 6:** Find the indefinite integral.

a.) $\int (3x+1)^4 dx$

$u = 3x+1$

$du = 3dx$

$\frac{1}{3}du = dx$

$\int u^4 \left(\frac{1}{3}du\right)$

$\frac{1}{3}\int u^4 du$

$\frac{1}{3}\left(\frac{1}{5}u^5\right) \rightarrow \frac{1}{15}u^5 + C \rightarrow \boxed{\frac{1}{15}(3x+1)^5 + C}$

b.) $\int e^{2x+5} dx$

$u = 2x+5$

$du = 2dx$

$\frac{1}{2}du = dx$

$\int e^u \left(\frac{1}{2}du\right)$

$\frac{1}{2}\int e^u du$

$\frac{1}{2}e^u + C \rightarrow \boxed{\frac{1}{2}e^{2x+5} + C}$

Hmwk E

**Example 6 Continued:**

c.) $\int \frac{3}{4x-2}\,dx$

$u = 4x-2$
$du = 4\,dx$
$\frac{1}{4}du = dx$

$3\int \frac{1}{u}dx$

$3\int \frac{1}{u}\left(\frac{1}{4}du\right)$

$\frac{3}{4}\int \frac{1}{u}du \rightarrow \frac{3}{4}\ln|u| \rightarrow \frac{3}{4}\ln|4x-2| + C$

d.) $\int \frac{1}{(6x+3)^4}\,dx$

$u = 6x+3$
$du = 6\,dx$
$\frac{1}{6}du = dx$

$\int \frac{1}{u^4}\left(\frac{1}{6}du\right)$

$\frac{1}{6}\int u^{-4}du$

$\frac{1}{6}\left(-\frac{1}{3}u^{-3}\right) \rightarrow -\frac{1}{18}(6x+3)^{-3}$

$\boxed{\frac{-1}{18(6x+3)^3} + C}$

**Example 7:** Find the indefinite integral.

mini example: $\int \underline{2x}\, e^{x^2} \underline{dx}$

$u = x^2$
$du = 2x\,dx$

$\int e^{x^2}\underline{2x\,dx} = \int e^u du = e^u = \boxed{e^{x^2} + C}$

a.) $\int (3x^2+5x)^4(6x+5)dx$

$u = 3x^2+5x$
$du = (6x+5)dx$

$\int u^4 du$

$\frac{1}{5}u^5 + C$

$\frac{1}{5}(3x^2+5)^5 + C$

b.) $\int \sqrt[3]{x^2-3x}\,(2x-3)dx$

$u = x^2-3x$
$du = (2x-3)dx$

$\int u^{\frac{1}{3}}du$

$\frac{3}{4}u^{4/3} + C$

$\frac{3}{4}(x^2-3x)^{4/3} + C$

c.) $\int xe^{4x^2+1}\,dx$

$u = 4x^2+1$
$du = 8x\,dx$
$\frac{1}{8}du = x\,dx$

$\int e^u \frac{1}{8}du$

$\frac{1}{8}\int e^u du$

$\frac{1}{8}e^u + C \rightarrow \boxed{\frac{1}{8}e^{4x^2+1} + C}$

d.) $\int \frac{\cancel{12x^3-3x^2}\ 24x^3-6x^2}{3x^4-x^3}\,dx$

$u = 3x^4-x^3$
$du = (12x^3-3x^2)dx$
$2du = (24x^3-6x^2)dx$

$\int \frac{1}{3x^4-x^3}(24x^3-6x^2)dx$

$\int \frac{1}{u}2du \Rightarrow 2\int \frac{1}{u}du$

$2\ln|u| + C$

$\boxed{2\ln|3x^4-x^3| + C}$

thank F