9.1 Antiderivatives and the indefinite integral

Definition: The function F is called an **antiderivative** of f if $F'(x) = f(x)$

Definition: The process of finding the antiderivative is called **antidifferentiation**.

Definition: If $F'(x) = f(x)$, we write $\int f(x)dx = F(x) + C$, where the expression $\int f(x)dx$ is called an **indefinite integral.** This statement is read "the antiderivative (or ingetral) of f with respect to x". *NOTE: It's important to realize "dx" is the **variable of integration** and determines the variable of which you are deriving.

Definition: When evaluating an indefinite integral, do not forget the **constant of integration (+C)**!

Example 1: Find the antiderivative of each function.

a.) x^{10}

$\frac{1}{11}x^{11} + C$

b.) $\frac{1}{x^5}$ $\quad x^{-5}$

$-\frac{1}{4}x^{-4} + C$

c.) $\sqrt[3]{x}$

d.) $\sqrt[4]{x^3}$ $\quad x^{3/4}$

$\frac{4}{7}x^{7/4} + C$

Integration Rules

- **Power Rule:** $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$, where n is not -1
- **Constant Rule:** $\int k dx = kx + C$
- **Constant Multiple Rule:** $\int kf(x)dx = k\int f(x)dx$
- **Sum or Difference Rule:** $\int \big(f(x) \pm g(x)\big)dx = \int f(x)dx \pm \int g(x)dx$

Example 2: Find the indefinite integral.

a.) $\int x^6 dx$

$\frac{1}{6+1}x^{6+1} = \frac{1}{7}x^7 + C$

b.) $\int 4dt$

$4t + C$

c.) $\int 3x^5 dx$

$3 \cdot \int x^5$

$3\left(\frac{1}{6}x^6\right) = \frac{1}{2}x^6 + C$

d.) $\int (3u^4 + 6u^2 + 2)du$

$3\int u^4 du + 6\int u^2 du + \int 2du$

$3\left(\frac{1}{5}u^5\right) + 6\left(\frac{1}{3}u^3\right) + 2u + C$

$\frac{3}{5}u^5 + 2u^3 + 2u + C$

e.) $\int (x + \sqrt[3]{x})dx$

$\int x + \int x^{1/3}$

$\frac{1}{2}x^2 + \frac{1}{\frac{1}{3}+\frac{3}{3}}x^{\frac{1}{3}+\frac{3}{3}} = \frac{1}{2}x^2 + \frac{3}{4}x^{4/3} + C$

Such as at $t=3$ $f(3)=60$

Definition: Sometimes a boundary condition is given as an **initial condition**, which represents a condition for when time is zero. We can use this information to write our **general solution** (one with +C) as a **particular solution** instead.

Example 3(a): If $f\,'(x) = 3x^2 + 2x$ and $f(2) = -3$, find $f(x)$.

$f(x) = 3\left(\frac{1}{3}x^3\right) + 2\left(\frac{1}{2}x^2\right) + C$

$f(x) = x^3 + x^2 + C$

$-3 = (2)^3 + (2)^2 + C$

$-3 = 8 + 4 + C$

$-3 = 12 + C$ $\quad C = -15$

$\boxed{f(x) = x^3 + x^2 - 15}$

Example 3(b): The curve $y = f(x)$ passes through the point (32,30). The gradient of the curve is given by $f\,'(x) = \frac{1}{\sqrt[5]{x^3}}$. Find the equation of the curve.

$f\,'(x) = x^{-3/5}$

$f(x) = \frac{1}{-\frac{3}{5}+\frac{5}{5}} x^{-3/5+5/5}$

$f(x) = \frac{5}{2}x^{2/5} + C$

$30 = \frac{5}{2}(32)^{2/5} + C$

$30 = (5/2)(4) + C$

$30 = 10 + C$ $\quad C = 20$

$\boxed{f(x) = \frac{5}{2}x^{2/5} + 20}$

Example 3(c): The rate of growth of a population of fish is given by $dP/dt = 150\sqrt{t}$ for $0 \le t \le 5$ years. The initial population was 200 fish. Find the number of fish at $t = 4$ years.

$\frac{dP}{dt} = 150t^{1/2}$

$f(x) = 150\left(\frac{1}{1/2+2/2} \cdot t^{\frac{1}{2}+\frac{2}{2}}\right)$

$150\left(\frac{2}{3} \cdot t^{3/2}\right)$

$f(x) = 100t^{3/2} + C$

$200 = 100(0)^{3/2} + C$

$C = 200$

$f(4) = 100(4)^{3/2} + 200$

$100(8) + 200$

$= 1000$ fish