9.1 Antiderivatives and the indefinite integral

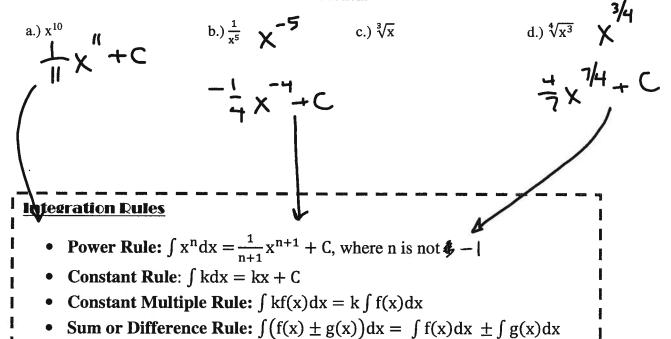
Definition: The function F is called an **antiderivative** of f if F'(x) = f(x)

Definition: The process of finding the antiderivative is called <u>antidifferentiation</u>.

Definition: If F'(x) = f(x), we write $\int f(x)dx = F(x) + C$, where the expression $\int f(x)dx$ is called an <u>indefinite integral</u>. This statement is read "the antiderivative (or ingetral) of f with respect to x". *NOTE: It's important to realize "dx" is the <u>variable of integration</u> and determines the variable of which you are deriving.

Definition: When evaluating an indefinite integral, do not forget the <u>constant of integration</u> (+C)!

Example 1: Find the antiderivative of each function.



Example 2: Find the indefinite integral.

a.)
$$\int x^{6} dx$$

$$\int \frac{1}{6+1} x^{6+1} = \frac{1}{7} x^{7} + C$$

$$4 + C$$

$$3 \cdot \int x^{5} dx$$

$$3 \cdot \left(\frac{1}{6} x^{6}\right) = \frac{1}{2} x^{6} + C$$

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$$4 \cdot \int (3u^{4}$$

Definition: Sometimes a boundary condition is given as an <u>initial condition</u>, which represents a condition for when time is zero. We can use this information to write our <u>general solution</u> (one with +C) as a <u>particular solution</u> instead.

Example 3(a): If
$$f'(x) = 3x^2 + 2x$$
 and $f(2) = -3$, find $f(x)$.

$$f(x) = 3(\frac{1}{3}x^3) + 2(\frac{1}{2}x^2) + C$$

$$-3 = 8 + 4 + C$$

$$-3 = 12 + C$$

$$C = -15$$

Example 3(b): The curve y = f(x) passes through the point (32,30). The gradient of the curve is given by $f'(x) = \frac{1}{5\sqrt{x^3}}$. Find the equation of the curve.

$$f'(x) = x^{-3/5}$$

$$f(x) = \frac{1}{-3k+1/5}$$

$$30 = \frac{5}{2}(32)^{3/5} + C$$

$$50 = (5/2)(4) + C$$

$$50 = 10 + C$$

$$C = 20$$

$$C(x) = \frac{5}{2}x^{2/5} + C$$

$$F(x) = \frac{5}{2}x^{2/5} + 20$$

Example 3(c): The rate of growth of a population of fish is given by $dP/dt = 150\sqrt{t}$ for $0 \le t \le 5$ years. The initial population was 200 fish. Find the number of fish at t = 4 years.

$$f(x) = |50 t^{1/2} + C$$

$$f(x) = |50 (\frac{1}{1/2} \cdot t^{\frac{1}{2} + \frac{3}{2}})$$

$$150 (\frac{3}{3} \cdot t^{\frac{3}{2}})$$

$$C = 200$$

$$f(x) = |60 t^{3/2} + C$$

$$f(4) = |60 (4)^{3/2} + 200$$

$$|80 (8) + 200$$

$$= |600 f(8)$$