14.4 *Revisiting linear motion*

Reminder of kinematics:

Displacement function: s (t)

Velocity: $s'(t)$

Acceleration: $s''(t)$

Total distance traveled from t_1 to t_2 = $\int_{t_1}^{t_2} |v(t)|\,dt$

initially: $t = 0$

At rest: $v(t) = 0$

initially at rest: $v(0) = 0$

Moving right/up: $v(t) > 0$

Moving left/down: $v(t) < 0$

Speed: $|v(t)|$

Example 10: A particle moves along a horizontal line. The particle's displacement, in meters, from an origin O is given by s (t) = 5 - 2cos3t for time t seconds.

a.) Find the particle's velocity and acceleration at any time t.

$s'(t) = v(t)$

$-2(-\sin(3t))\cdot 3$

$v(t) = 6\sin 3t$

$v'(t) = a(t)$

$6(\cos(3t))\cdot 3$

$\boxed{a(t) = 18\cos 3t}$

b.) Find the particle's initial displacement, velocity, and acceleration.

$s(0) = 5 - 2\cos 3(0)$

$= 5 - 2\cos 0$

$5 - 2(1)$

$\boxed{s(0) = 3\text{ m}}$

(1,0) cos, sin

$v(0) = 6\sin 3(0)$

$= 6\sin 0$

$\boxed{v(0) = 0\text{ m/s}}$

$a(0) = 18\cos 3(0)$

$= 18\cos 0$

$= 18(1)$

$\boxed{a(0) = 18\text{ m/s}^2}$

c.) Find when the particle is moving to the right, to the left, and stopped during the time [0,π].

$6\sin 3t = 0$

$\sin 3t = 0$

(1,0)

$3t = 0 \quad 3t = \pi \quad 3t = 2\pi \quad 3t = 3\pi$

$t = 0,\ \pi/3,\ \frac{2\pi}{3},\ \pi$

Stopped at

$v(t)$: 0 (+) $\frac{\pi}{3}$ (−) $\frac{2\pi}{3}$ (+) π

Right $(0, \pi/3) \cup (2\pi/3, \pi)$

Left $(\pi/3, 2\pi/3)$

d.) Write down a definite integral that represents the total distance traveled for [0,π] seconds and use a GDC to find the distance.

$\int_0^{\pi} |v(t)|\,dt \rightarrow \int_0^{\pi} |6\sin 3t|\,dt = 12$

$\boxed{\text{or}}\ \left|\int_0^{\pi/3} v(t)\,dt\right| + \left|\int_{\pi/3}^{2\pi/3} v(t)\,dt\right| + \left|\int_{2\pi/3}^{\pi} v(t)\,dt\right|$

$|4| + |-4| + |4| = 12$

Example 11: A particle moves along a straight line so that its velocity, v ms^{-1} at time *t* seconds is given by *v* (t) = 5sin t $\cos^2$ t.

S A
T C   Ref $\pi/6$

a.) Find the speed of the particle when t = 5π/6 seconds.

$$\text{Speed} = |v(t)| = |v(5\pi/6)| = 5\sin\frac{5\pi}{6}\left(\cos\frac{5\pi}{6}\right)^2$$

$$5\left(\tfrac{1}{2}\right)\left(-\sqrt{3}/2\right)^2$$

$$5\left(\tfrac{1}{2}\right)\left(3/4\right)$$

$$\boxed{15/8 \ \ m/s}$$

b.) When t = 0, the displacement, *s*, of the particle is 3m. Find an expression for *s* in terms of *t*.

$s(0) = 3$

$$s(t) = \int 5\sin t(\cos t)^2\,dt$$

$u = \cos t$

$du = -\sin t\,dt$

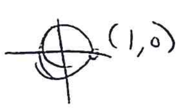
$-du = \sin t\,dt$

$$= 5\int u^2 \sin t\,dt$$

$$-5\int u^2\,du$$

$$-5\cdot\tfrac{1}{3}\cos^3 t + C$$

$$\boxed{s(t) = -\tfrac{5}{3}\cos^3 t + \tfrac{14}{3}}$$



$$-\tfrac{5}{3}(\cos 0)^3 + C = 3$$

$$-5/3(1)^3 + C = 3 \qquad \boxed{C = 14/3}$$

c.) Find an expression for the acceleration, *a*, of the particle in terms of *t*.

$a(t) = v'(t)$

$v(t) = 5\sin t\cos^2 t$ product & chain rule

$$v'(t) = 5\left[\cos t\cdot\cos^2 t + \sin t\cdot 2\cos t\cdot -\sin t\right]$$

$$5\left[\cos^3 t - 2\cos t\sin^2 t\right]$$

$$\boxed{a(t) = 5\cos t\left[\cos^2 t - 2\sin^2 t\right]}$$