14.3 Integral of sine and cosine (2 days)

Reminder of Integral Rules:

- **Power Rule:** $\int x^n\, dx = \frac{1}{n+1} x^{n+1} + C, n \neq 1$
- **Constant Rule:** $\int k\, dx = kx + C$
- **Constant Multiple Rule:** $\int k f(x) dx = k \int f(x) dx$
- **Sum or Difference Rule:** $\int [f(x) \pm g(x)] dx = \int f(x)\, dx \pm \int g(x) dx$
- **Integral of 1/x:** $\int \frac{1}{x} dx = \ln x + C, x > 0$
- **Integral of e^x:** $\int e^x dx = e^x + C$
- **Integral of sin x:** $\int \sin x\, dx = -\cos x + C$
- **Integral of cos x:** $\int \cos x\, dx = \sin x + C$
- *Careful when you have a function in terms of another function to use substitution!*

Example 6: **Find the integrals.**

a.) $\int 3 \sin x\, dx$

b.) $\int \cos(4x - 6) dx$

c.) $\int e^x \sin(e^x)\, dx$

d.) $\int x^3 \cos(3x^4)\, dx$

Reminder of the Fundamental Theorem of Calulus:

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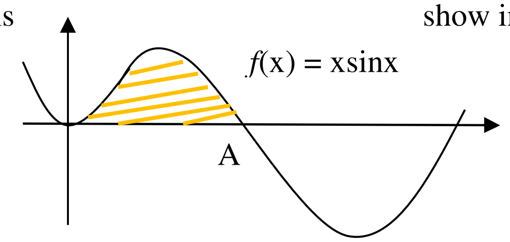
Example 7: Evaluate the definite integral without a GDC to get the exact value. Then check your answer by evaluating the definite integral on the GDC.

a.) $\int_0^{\pi/4} 2\cos x\, dx$

b.) $\int_{\pi/4}^{\pi/2} \sin(2x)\cos^3(2x)dx$

Remember that we can use a definite integral to represent an area bounded by a curve and the x-axis and can also find the volume obtained by rotating the object 360°. (See 9.6 Notes for help.)

Example 8: A portion of the graph of $f(x) = x \sin x$ is show in the diagram on page 509.



a.) Find the area of the shaded region. (use a GDC).

b.) Write down the integral representing the volume of the solid formed when the shaded region is rotated 360° about the x-axis. Hence, find the volume of the solid. (You can use a GDC).

Also remember that we can find the area between two curves.
If $y_1 \geq y_2$ for all x in [a,b], then $\int_a^b (y_1 - y_2)dx$ is the area between the two curves. (TOP - BOTTOM)

Example 9: Find the area of the region in quadrant 1 that is bounded by the curves y = 0.4x and y = sin x. (You may use a GDC).