14.2 More Practice with Derivatives

Reminder of Derivative Rules:

Old Rules	*New Rules*
d/dx[x^n] = $n x^{n-1}$, n ≠1	d/dx[sin x] = $\cos x$
d/dx[e^x] = e^x	d/dx[cos x] = $-\sin x$
d/dx[ln x] = $1/x$, x > 0	d/dx[tan x] = $1/\cos^2 x = \sec^2 x$

Example 3: Find the derivative of each function.

a.) $f(x) = 4e^{2x} + \sin(3x + 2)$ Chain rule

$$4e^{2x} \cdot 2 + \cos(3x+2) \cdot 3$$

$$\boxed{y' = 8e^{2x} + 3\cos(3x+2)}$$

b.) $y = e^x \sin x$ product rule

$$e^x \cdot \sin x + \cos x \cdot e^x$$

$$y' = e^x(\sin x + \cos x)$$

c.) $y = \cos^3 x \sin x$ product rule Chain rule

$$3\cos^2 x \cdot (-\sin x) \sin x +$$
$$\cos x \cdot \cos^3 x$$

$$\boxed{y' = -3\sin^2 x \cos^2 x + \cos^4 x}$$

d.) $s(t) = \ln(\sin t)$ chain rule

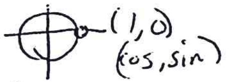
$$y' = \frac{1}{\sin t} \cdot \cos t$$

$$y' = \frac{\cos t}{\sin t}$$

$$\boxed{y' = \cot t}$$

Hmwk C

Example 4: Consider the function $f(x) = \sin x + \cos x$ for [0, 2π]. Analyze it without using a GDC.



a.) Find the x- and y-intercepts.

$$f(0) = \begin{matrix} \sin 0 + \cos 0 \\ 0 + 1 \end{matrix} = 1$$

x int

$$0 = \sin x + \cos x$$
$$\sin x = -\cos x$$

$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$ Reference $\pi/4$

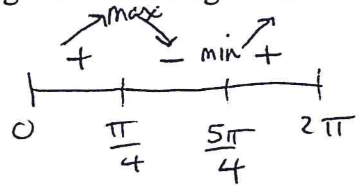
$\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

b.) Find the intervals on which f is increasing and decreasing and the relative extreme points.

$$f'(x) = \cos x + -\sin x$$
$$\cos x - \sin x = 0$$
$$\cos x = \sin x$$

$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

Ref $\pi/4$ $\pi/4 +$ $5\pi/4$



increasing
$(0, \pi/4) \cup (\frac{5\pi}{4}, 2\pi)$

decreasing
$(\pi/4, 5\pi/4)$

$$f(\pi/4) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$ max $(\pi/4, \sqrt{2})$

$$f(5\pi/4) = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$ min $(5\pi/4, -\sqrt{2})$

$\nearrow f'(x) = \cos x - \sin x$

Example 4 cont.: Consider the function *f* (x) = sin x + cos x for [0, 2π]. Analyze it without using a GDC.

c.) Find the intervals on which *f* is concave up and concave down and the inflexion points.

$f''(x) = -\sin x - \cos x$

$0 = -\sin x - \cos x$

$\frac{S \mid A}{T \mid C}$ $\sin x = -\cos x$

Ref π/4 $\boxed{3\pi/4,\ 7\pi/4}$

− + −

0 $\frac{3\pi}{4}$ $\frac{7\pi}{4}$ 2π

con ↑ $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right)$

con ↓ $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$

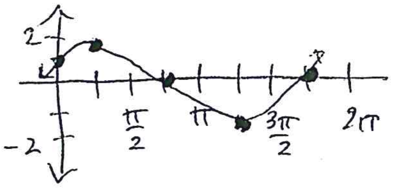
$f(3\pi/4) = 0$

$f(7\pi/4) = 0$

Inf. Pts

$\left(\frac{3\pi}{4}, 0\right) \left(\frac{7\pi}{4}, 0\right)$

d.) Use the information from parts **a** through **c** to sketch the graph *f*.



y-int (0,1)

x-int $\left(\frac{3\pi}{4}, 0\right)\left(\frac{7\pi}{4}, 0\right)$

max $(\pi/4, \sqrt{2})$

min $\left(\frac{5\pi}{4}, -\sqrt{2}\right)$

Example 5(a): Show how to use the second derivative test to find the x-coordinates of the relative extrema of *f* (x) = ln x + sin x on [0,2π].

$f'(x) = \frac{1}{x} + \cos x$

$0 = \frac{1}{x} + \cos x$

use calc Graph, calc, zeros

$x \doteq 2.074$

$x \doteq 4.488$

$f'(x) = x^{-1} + \cos x$

$f''(x) = -x^{-2} - \sin x$

$f''(x) = \frac{-1}{x^2} - \sin x$

$f''(2.074) = -1.109$ max

$f''(4.488) = +0.925$ min

$\boxed{\text{or}}$

max ↗ ↘ min ↗

+ − +

0 2.074 4.488 2π

$f(2.074) \doteq 1.606$

max (2.074, 1.606)

$f(4.488) \doteq .526$

min (4.488, .526)

Example 5 (b): Find the global extrema of the function *f* (x) = x + sin (x²) on the interval [0,π].

(absolute)

$f'(x) = 1 + \cos(x^2) \cdot 2x$

$f'(x) = 1 + 2x\cos(x^2)$

$0 = 1 + 2x\cos(x^2)$

use calc

$x \doteq 1.392$

$x \doteq 2.115$

$x \doteq 2.834$

$f(0) = 0$ ← Ab. min

$f(\pi) \doteq 2.711$

$f(1.392) \doteq 2.325$

$f(2.115) \doteq 1.143$

$f(2.834) \doteq 3.818$ ← Ab. max

$\boxed{\text{Hmwk D}}$