Name: _____ Class: _____ Date: _____ ID: A

IB Math SL Second Semester Final Exam Review

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. If $f(x) = x^2 - 2x + 1$, evaluate the difference quotient $\frac{f(a+h) - f(a)}{h}$.

Select the correct answer.
a. h b. $2a - 2$ c. $2a - h - 2$ d. $2a + h - 2$ e. none of these

____ 2. Evaluate the function $f(x) = 2\frac{\sqrt{x} - 1}{x - 1}$ at the given numbers (correct to six decimal places). Use the results to guess the value of $\lim_{x \to 1} f(x)$.

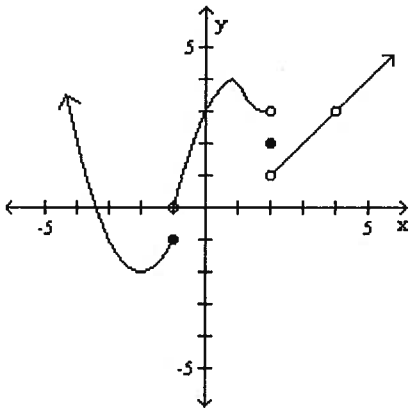
x	0.6	0.8	0.9	0.99	0.999
$f(x)$	1.127017	1.055728	1.026334	1.002513	1.00025

table cont'd

x	1.4	1.2	1.1	1.01	1.001
$f(x)$	0.91608	0.954451	0.976177	0.997512	0.99975

Select the correct answer.
a. 1 b. 0.899772 c. 0.98073 d. 0.828162 e. 0.691856

____ 3. **Use the graph of y = f(x) to answer the questions below.**



a. $\lim_{x \to 1} f(x)$ b. $\lim_{x \to 2^+} f(x)$ c. $\lim_{x \to 2} f(x)$ d. $f(2)$ e. $\lim_{x \to 4} f(x)$ f. $f(4)$ g. $\lim_{x \to \infty} f(x)$ h. $\lim_{x \to -\infty} f(x)$

____ 4. Evaluate the limit.

$$\lim_{x \to 1} (x+2)^3 (x^2 - 6)$$

Select the correct answer.
a. 27 b. −45 c. −135 d. 29 e. −125

____ 5. If $\lim_{x \to 2^-} f(x) = 4.5$, then if $\lim_{x \to 2} f(x)$ exists, to what value does it converge?

Select the correct answer.
a. 2 b. 1 c. 5 d. 4.5 e. 1.5

____ 6. Evaluate the limit..

$$\lim_{x \to 3} \frac{x^2 + 6x - 27}{x - 3}$$

Select the correct answer.
a. 15 b. 12 c. 16 d. 11 e. 10

____ 7. Find the limit.

$$\lim_{x \to \infty} \frac{x^7 - 3}{x^6 + 6}$$

Select the correct answer.
a. ∞ b. −∞ c. 0 d. 7/6 e. −1/2

Find the limit.

____ 8. $\lim_{x \to \infty} \left(\frac{7x}{x-6} + \frac{3x}{x+6} \right)$

a. 126 b. 10 c. 42 d. ∞

____ 9. Find the slope of the tangent line to the curve $y = 3x^2$ at the point (−3, 27).

Select the correct answer.
a. −18 b. 81 c. 27 d. −3 e. 18

____ 10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

Select the correct answer.

a. $y - 1 = -\left(x - \frac{\pi}{4}\right)$ b. $y - 1 = -2\left(x - \frac{\pi}{4}\right)$ c. $y = 2\left(x - \frac{\pi}{4}\right)$ d. $y = -\left(x - \frac{\pi}{4}\right)$ e. $y = -2\left(x - \frac{\pi}{4}\right)$

____ 11. Find the points on the curve $y = 2x^3 + 3x^2 - 36x + 7$ where the tangent is horizontal.

Select the correct answer.

a. (–4, 71), (4, 39) b. (–3, 88), (4, 39) c. (–4, 71), (2, 37) d. (–3, 88), (2, –37) e. (–3, 37), (2, –37)

____ 12. Differentiate.

$K(x) = (2x^3 + 1)(x^4 - 2x)$

Select the correct answer.

a. $6x^2 + 4x^3 - 2$ b. $14x^6 + 4x^3 - 12x^2 - 2x$ c. $x^2(x^4 + 2x) + (2x^3 + 1)(4x^3 + 2)$

d. $6x^2(x^4 - 2) + 4x^3(2x^3 + 1)$ e. $6x^2(x^4 - 2x) + (2x^3 + 1)(4x^3 - 2)$

____ 13. If $y = 4e^{9x}$, find y'.

Select the correct answer.

a. $36e^{9x}$ b. $4xe^{36x}$ c. $4e^{36x}$ d. $36xe^{9x}$

____ 14. If $y = x\ln\left(x^2\right)$, find $\frac{dy}{dx}$.

Select the correct answer.

a. $\ln\left(x^2\right) + 1$ b. $\ln\left(x^2\right) + 2$ c. $\ln\left(x^2\right) + \frac{1}{x}$ d. $\frac{1}{x^2}$ e. $\frac{1}{x}$

____ 15. If $f(t) = \sqrt{4t + 1}$, find $f''(2)$.

Select the correct answer.

a. $-\frac{4}{27}$ b. 3 c. $-\frac{2}{3}$ d. $\frac{2}{3}$ e. $\frac{4}{27}$

____ 16. The position function of a particle is given by $s = t^3 - 1.5t^2 - 2t,\ t \geq 0$.

When does the particle reach a velocity of 166 m/s?

Select the correct answer.

a. $t = 7$ sec b. $t = 8$ sec c. $t = 5$ sec d. $t = 3$ sec e. $t = 12$ sec

____ 17. The equation of motion is given for a particle, where s is in meters and t is in seconds. Find the acceleration after 4.5 seconds.

$s = \sin 2\pi t$

Select the correct answer.
a. 9π m/s^2 b. -9π m/s^2 c. 0 m/s^2 d. $81\pi^2$ m/s^2 e. $-81\pi^2$ m/s^2

____ 18. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters seconds later is given by $y = 10t - 1.86t^2$. Find the average velocity over the time interval [1, 2].

Select the correct answer.
a. 4.42 m/s b. –3.72 m/s c. 3.72 m/s d. 2.21 m/s

____ 19. What is the instantaneous rate of change at $t = -1$ of the function f given by $f(t) = \dfrac{t^3 + t}{4t + 1}$?

Select the correct answer.
a. $\dfrac{12}{9}$ b. $\dfrac{4}{9}$ c. $-\dfrac{20}{9}$ d. $-\dfrac{4}{9}$ e. $\dfrac{2}{3}$

____ 20. A mass on a spring vibrates horizontally on a smooth level surface. Its equation of motion is $x(t) = 3\sin t$ where t is in seconds and x in centimeters. Find the velocity at time t.

Select the correct answer.
a. $v(t) = \cos 3t$ b. $v(t) = \sin 3t$ c. $v(t) = 3\sin 3t$ d. $v(t) = 3\cos t$ e. $v(t) = 2\cos 3t$

____ 21. A particle moving along the x-axis such that at any time $t \geq 0$ its position is given by $x(t) = 2 - 30t + 81t^2 - 20t^3$. For what values of t is the particle moving to the left?

Select the correct answer.
a. $0 \leq t < \dfrac{27}{20}$ b. $t > \dfrac{27}{20}$ c. $0 \leq t < \dfrac{1}{5}, t > \dfrac{5}{2}$ d. $\dfrac{1}{5} < t < \dfrac{5}{2}$ e. $t > \dfrac{1}{5}$

____ 22. Find the critical numbers of the function.

$y = 5x^2 + 20x$

Select the correct answer.
a. 5, 0 b. –2 c. 20 d. 0 e. –1

____ 23. How many points of inflection are on the graph of the function?

$f(x) = 18x^3 + 5x^2 - 12x - 17$

Select the correct answer.
a. 1 b. 2 c. 4 d. 3 e. 5

____ 24. The function defined by $f(x) = x^3 - 3x^2$ for all real numbers x has a relative maximum at $x =$

Select the correct answer.
a. –2 b. 0 c. 1 d. 2 e. 4

____ 25. The function f given by $f(x) = x^3 + 12x - 24$ is

Select the correct answer.
a. increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
b. decreasing for $x < 0$, increasing for $x > 0$ c. increasing for all x d. decreasing for all x
e. decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

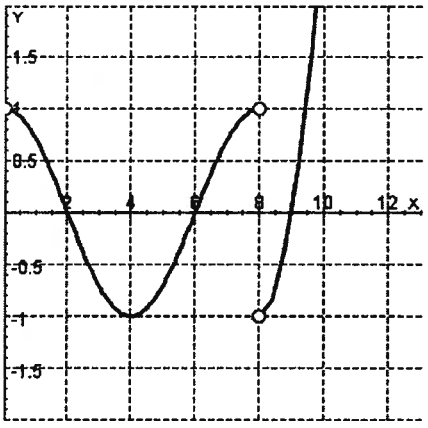
____ 26. The graph of $f(x) = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for?

Select the correct answer.
a. $x > 0$ b. $x < 0$ c. $x < -2$ and $x > -\frac{2}{3}$ d. $x < \frac{2}{3}$ and $x > 2$ e. $\frac{2}{3} < x < 2$

____ 27. Find the absolute maximum value of $y = -2x^2 - 16x - 33$ on the interval $[-5, 0]$.

Select the correct answer.
a. –4 b. –1 c. 0 d. 2 e. none

____ 28. The graph of the derivative $f'(x)$ of a continuous function f is shown. Answer the following:



i) On what intervals is f increasing? Why?

ii) At what x value(s) is there a local minimum of f? Why?

iii) What is happening on f at $x = 4$? Why?

iv) What is happening on f at $x = 8$? Why?

____ 29. We need to enclose a field with a fence. We have 500 feet of fencing material with a building on one side of the field where we will not need any fencing. Determine the maximum area of the field that can be enclosed by the fence.

a. 125 feet by 250 feet b. 150 feet by 200 feet c. 125 feet by 100 feet d. 200 feet by 150 feet

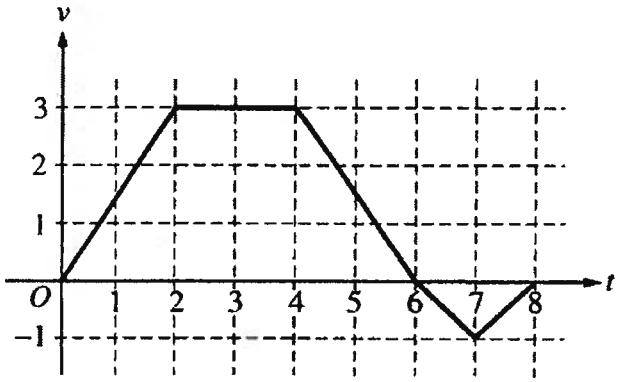
____ 30. A rectangular box with a square base and no top has a volume of 500 cubic inches. Find the length of the edge of the square base and height for the box that requires the least amount of material to build.

a. Length: 10 inches; Height: 10 inches
b. Length: 9.17 inches; Height: 6 inches
c. Length: 9 inches; Height: 6.17 inches
d. Length: 10 inches; Height: 5 inches

____ 31. A rectangular box with a square base with no top has a surface area of 108 square feet. Find the dimensions that will maximize the volume.

a. Length: 4.15 feet; Height: 4.85 feet
b. Length: 6.15 feet; Height: 2.85 feet
c. Length: 6 feet; Height: 3 feet
d. Length: 3.85 feet; Height: 6.15 feet

____ 32. A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown below



At what value of t does the bug change direction?

Select the correct answer.
a. 2 b. 4 c. 6 d. 7 e. 8

____ 33. Find the most general antiderivative of the function.

$f(x) = 18x^2 - 14x + 9$

Select the correct answer.
a. $F(x) = 30x^5 - 28x^4 + 9x + C$ b. $F(x) = 6x^3 - 7x^2 + 9x + C$ c. $F(x) = 18x^3 - 14x^2 + 9x + C$
d. $F(x) = 36x - 14 + C$ e. $F(x) = 6x^2 - 14x + C$

____ 34. Find f.

$f'(t) = 2t - 3\sin t, f(0) = 5$

Select the correct answer.
a. $f(t) = t^2 + 3\cos t + 2$ b. $f(t) = t^2 + 3\cos t$ c. $f(t) = 2t - 3\sin t + 5$ d. $f(t) = t^2 - 3\cos t - 5$

____ 35. Find f.

$f''(x) = 12x + 24x^2$

Select the correct answer.
a. $f(x) = 6x^3 + 4x^4 + Cx + D$ b. $f(x) = 2x^3 + 2x^4 + Cx + D$ c. $f(x) = 4x^3 + 8x^4 + Cx + D$
d. $f(x) = 2x^3 + x^4 + Cx + D$ e. $f(x) = x^3 + 4x^4 + Cx + D$

____ 36. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

Select the correct answer.
a. −10 b. 17 c. 3 d. −3 e. 2

____ 37. Evaluate the integral.

$$\int_{\pi/3}^{\pi/2} \sin t\,dt$$

Select the correct answer.
a. −1.000 b. 0.250 c. 1.000 d. −0.500 e. 0.500

____ 38. Evaluate the integral.

$$\int_0^3 (6 + 6y - y^2)\,dy$$

Select the correct answer.
a. −18 b. 45 c. 54 d. 36 e. −12

____ 39. Evaluate the indefinite integral.

$$\int \cos^7 x \sin x \, dx$$

Select the correct answer.

a. $\frac{1}{8}\cos^8 x + C$ b. $-\frac{1}{8}\sin^8 x + C$ c. $-\frac{1}{8}\cos^7 x + C$ d. $\frac{1}{8}\sin^8 x + C$ e. $-\frac{1}{8}\cos^8 x + C$

____ 40. Evaluate the integral.

$$\int x^2\sqrt{x^3+2}\, dx$$

Select the correct answer.

a. $-\frac{2}{9}(x^3+2)^{3/2} + C$ b. $\frac{1}{9}(x^3+2)^{1/2} + C$ c. $\frac{2}{9}(x^3+2)^{1/2} + C$ d. $\frac{2}{9}(x^3+3)^{3/2}$ e. $\frac{2}{9}(x^3+2)^{3/2} + C$

____ 41. Evaluate the integral if it exists.

$$\int_0^1 3x^2 \cos(x^3) dx$$

Select the correct answer.

a. sin(0.1) b. 3sin(0.1) c. 3sin(1) d. cos(1) e. none of these

____ 42. Evaluate the indefinite integral.

$$\int \frac{4+10x}{\sqrt{7+4x+5x^2}}\, dx$$

Select the correct answer.

a. $2\sqrt{7+4x+5x^2} + C$ b. $\sqrt{7+4x+5x^2} + C$ c. $-2\sqrt{7+4x+5x^2}$ d. $-2\sqrt{7+4x+5x^2} + C$
e. $3\sqrt{7+4x+5x^2} + C$

____ 43. Use a calculator to evaluate the integral.

$$\int_{-2}^{5} \left|4x - x^2\right| dx$$

Select the correct answer. The choices are rounded to the nearest hundredth.

a. 109.00 b. 123.67 c. 102.33 d. 209.00 e. 23.67

____ 44. The velocity function (in meters per second) for a particle moving along a line is $v(t) = 6t - 6$. Find the displacement of the particle at $t = 5$.

Select the correct answer.
a. 105 m b. −15 m c. 45 m d. 148.8 m e. 115 m

____ 45. Find the area of the region that lies beneath the given curve: $y = \sin x,\ 0 \le x \le \frac{\pi}{3}$

Select the correct answer.
a. 1.500 b. −1.500 c. 0.500 d. −0.500 e. 1.450

____ 46. Find the area of the region that lies under the given curve.

$y = \sqrt{2x+2},\ 0 \le x \le 1$

Select the correct answer.
a. 1.834 b. 1.727 c. 1.704 d. 1.724 e. 1.824

____ 47. Find the area of the region bounded by the curves.

$y = x^2 - 2x,\ y = x + 4$

Select the correct answer.
a. $\frac{125}{3}$ b. $\frac{25}{3}$ c. 5 d. 20 e. $\frac{125}{6}$

____ 48. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^2$ and $x = 9$ about the x-axis.

Select the correct answer.
a. 9π b. $\frac{243}{5}\pi$ c. $\frac{59049}{5}\pi$ d. 243π

IB Math SL Second Semester Final Exam Review
Answer Section

MULTIPLE CHOICE

1. D
2. A
3. a. 4 b. 1 c. DNE d. 2 e. 3 f. DNE g. ∞ h. ∞
4. C
5. D
6. B
7. A
8. B
9. A
10. E
11. D
12. E
13. A
14. B
15. A
16. B
17. C
18. A
19. D
20. D
21. C
22. B
23. A
24. B
25. C
26. E
27. B
28. i) $(0,2) \cup (6,8) \cup (9,\infty)$ → the graph is positive there
ii) $x=6$ and $x=9$ → the graph goes from negative to positive
iii) There is an inflection pt → the graph has a minimum
iv) The graph is not differentiable → the graph is undefined There.
29. A
30. D
31. C
32. C
33. B
34. A
35. B
36. C
37. E
38. D
39. E
40. E
41. E
42. A
43. E
44. C
45. C
46. D
47. E
48. C